DG-enhanced cyclotomic KLR algebras and categorification of Verma modules

Pedro Vaz (Université catholique de Louvain)

$$M^{\mathfrak{p}}(V_\beta) = U_q(\mathfrak{g}) \otimes_{U_q(\mathfrak{p})} V_\beta$$

Joint work with Grégoire Naisse and Ruslan Maksimau

September 2018

A well known story-I

Pick your favorite qKM algebra $\mathfrak{g}$ and let Λ be a dominant integral weight.

$L(\Lambda)$ Integrable irreducible
(fin. dim. if $\mathfrak{g}$ of finite type)

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$$\begin{array}{ccc} N(\Lambda) & \hookrightarrow & \Delta(\Lambda) \\ & & \downarrow \\ & & L(\Lambda) \end{array}$$

Verma

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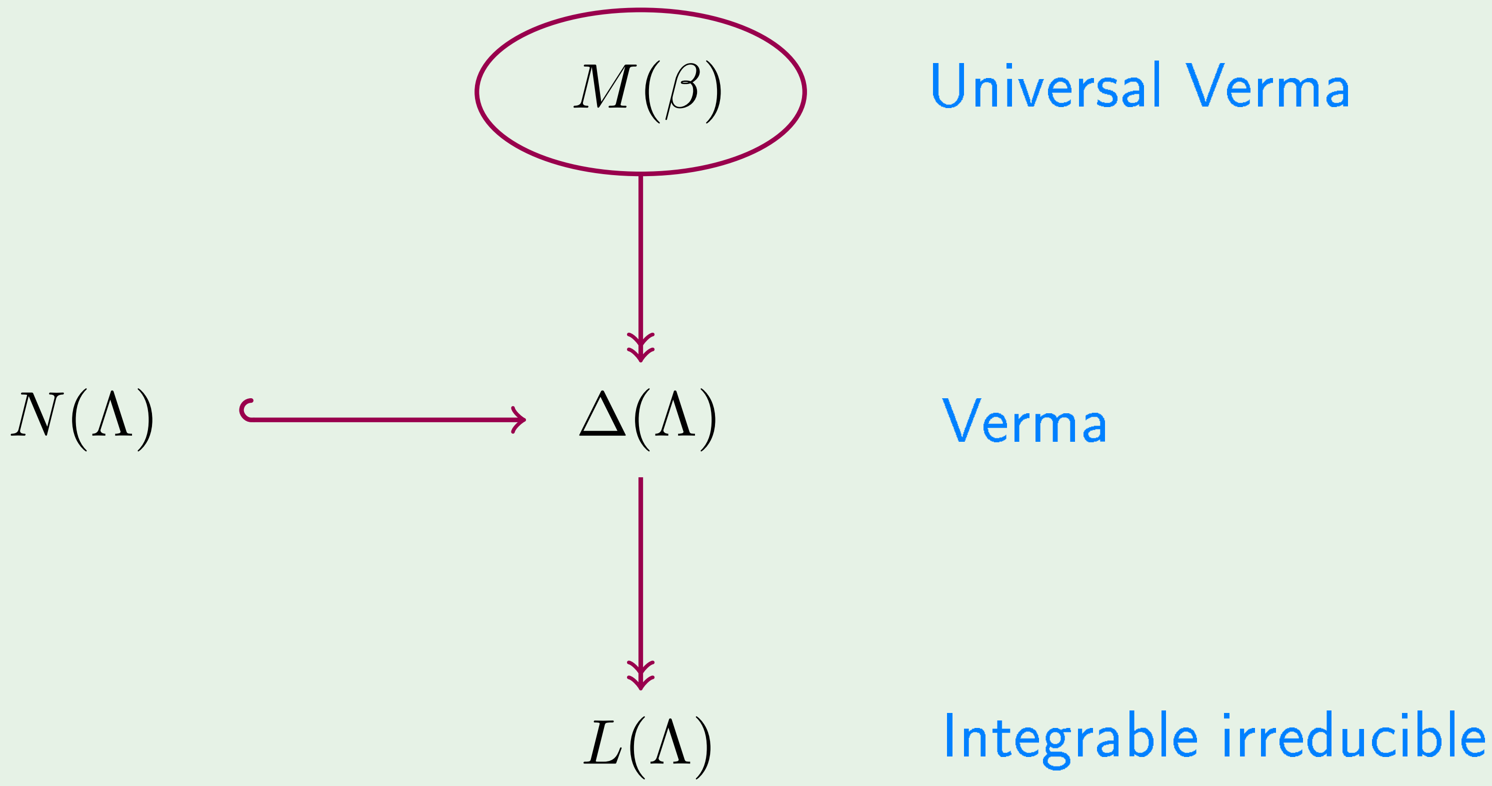

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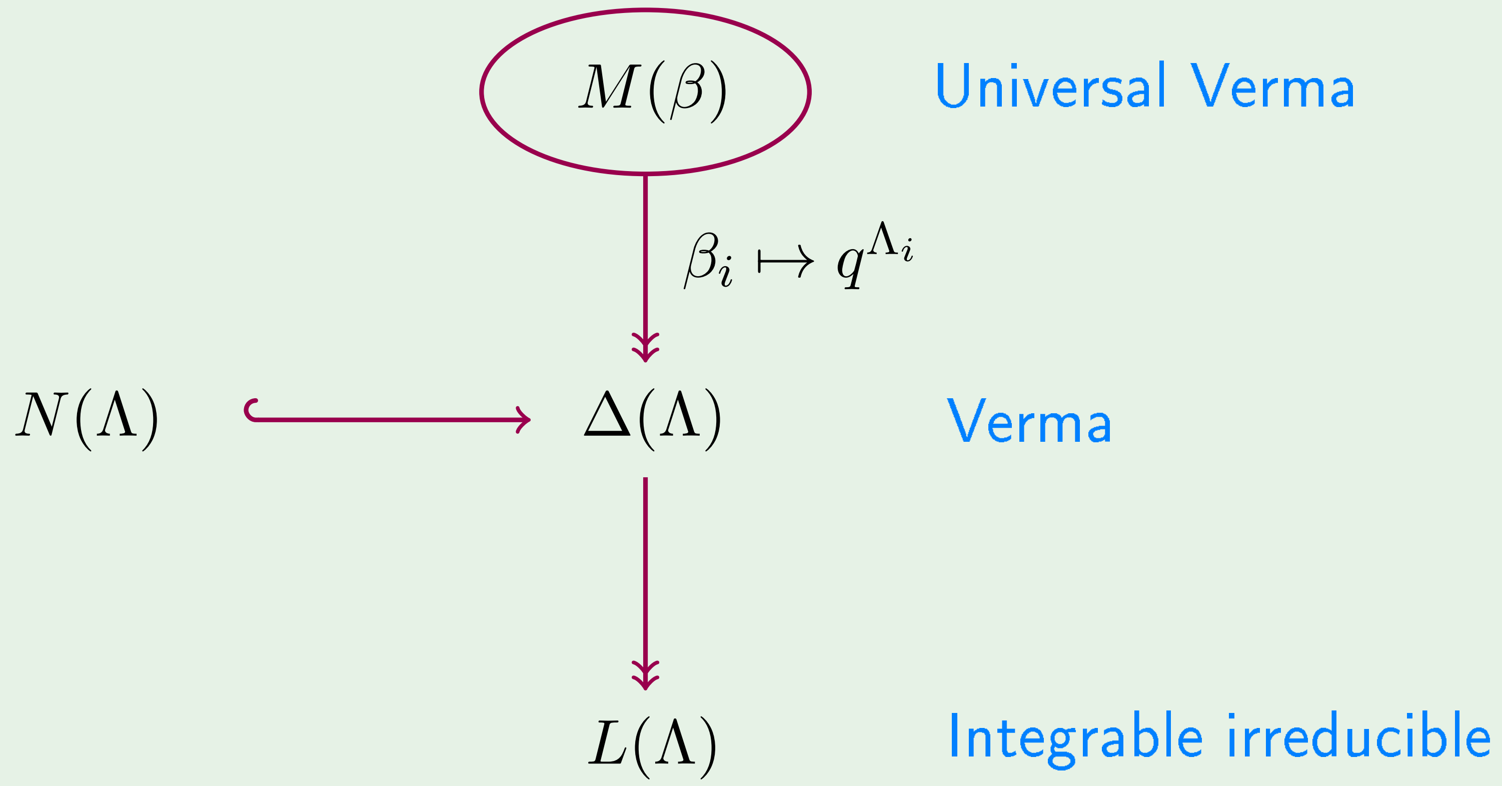

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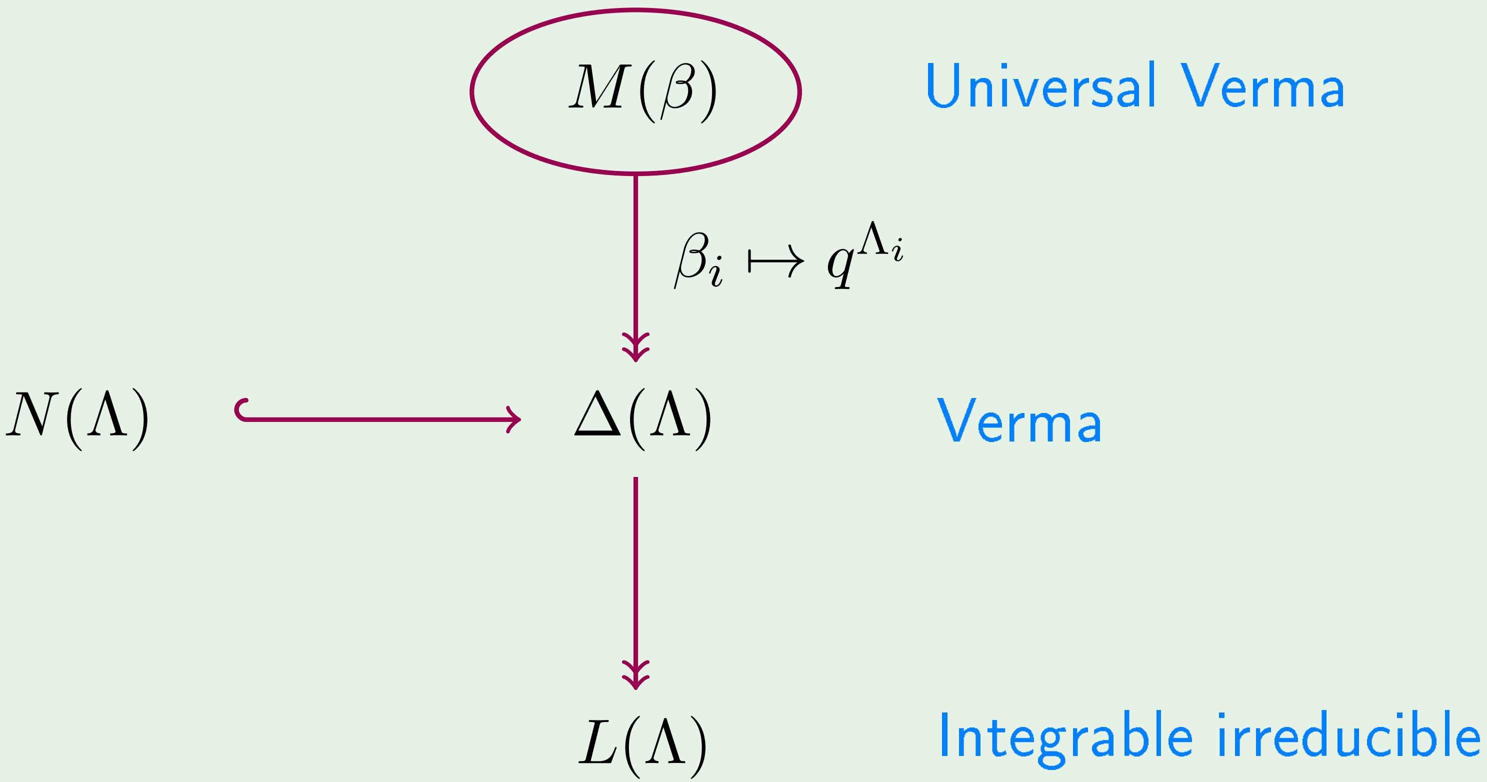

We think of these as modules over $\Bbbk(q)$ and $\Bbbk(q, \beta_1, \ldots, \beta_\ell)$.

A well known story-II

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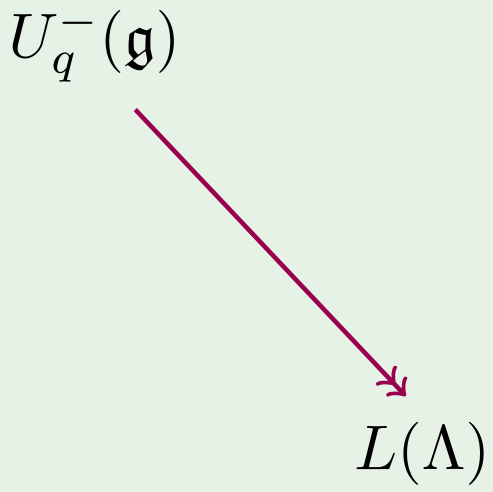

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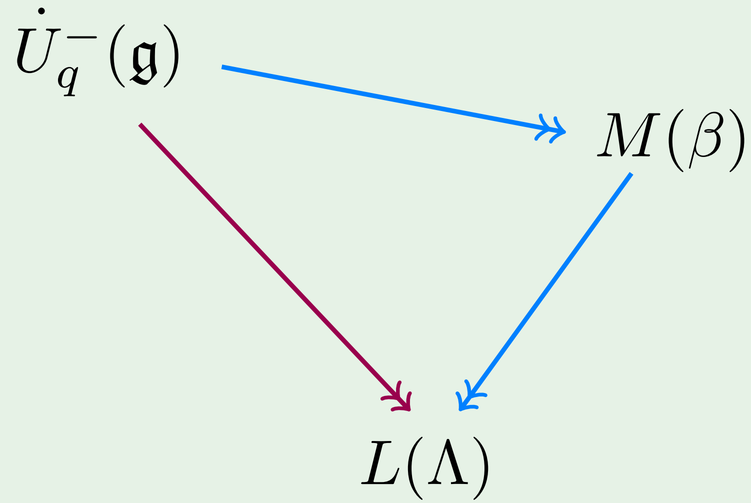

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$$\begin{array}{ccccc}
\mathcal{U}_q^-(\mathfrak{g}) & \xrightarrow{K_0} & U_q^-(\mathfrak{g}) & & \\
& \searrow & & \searrow & \\
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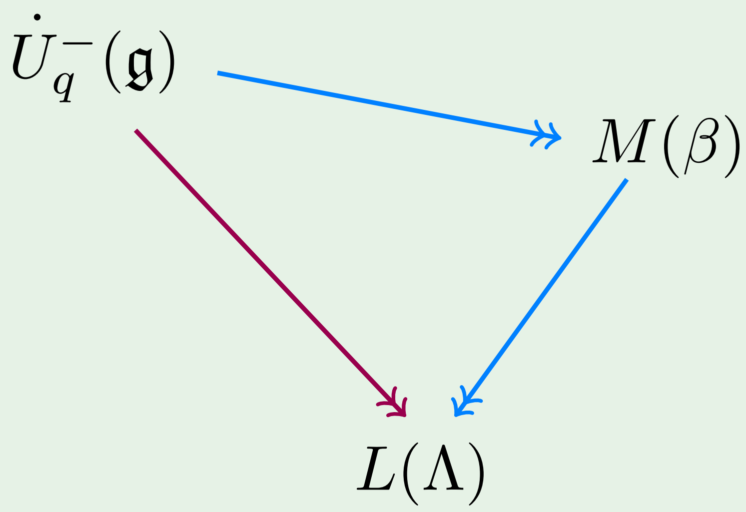

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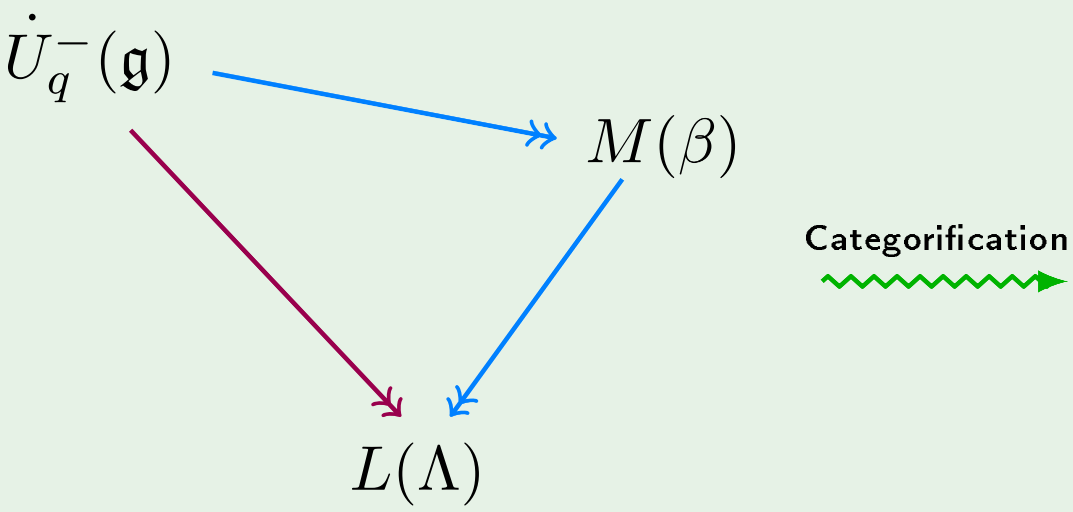

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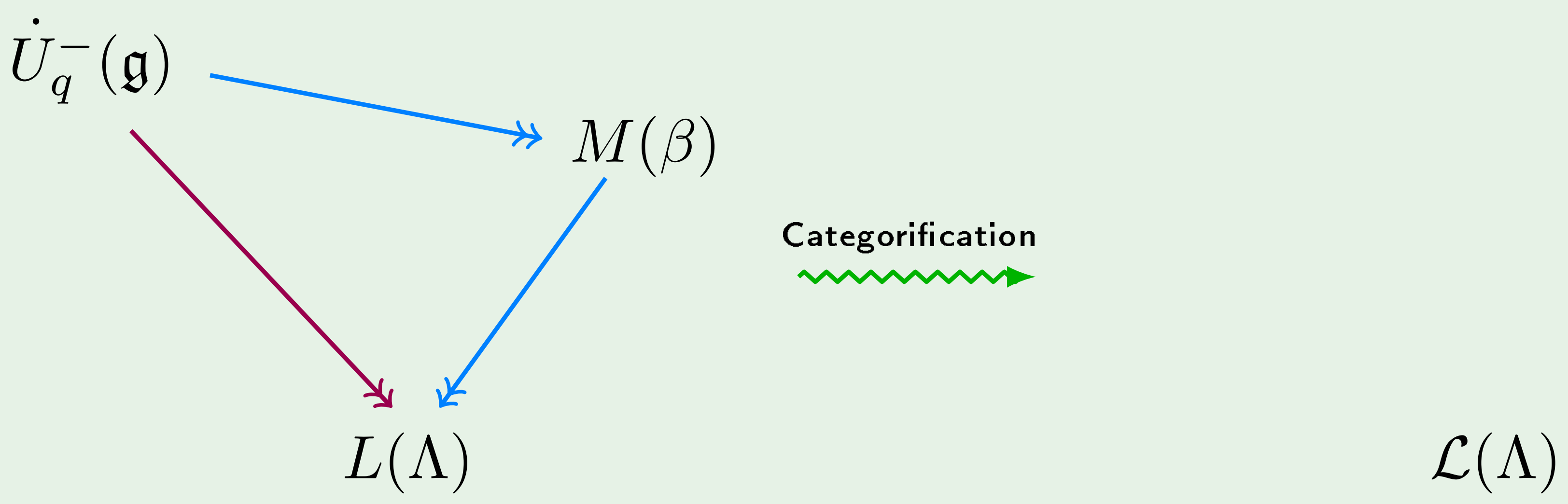

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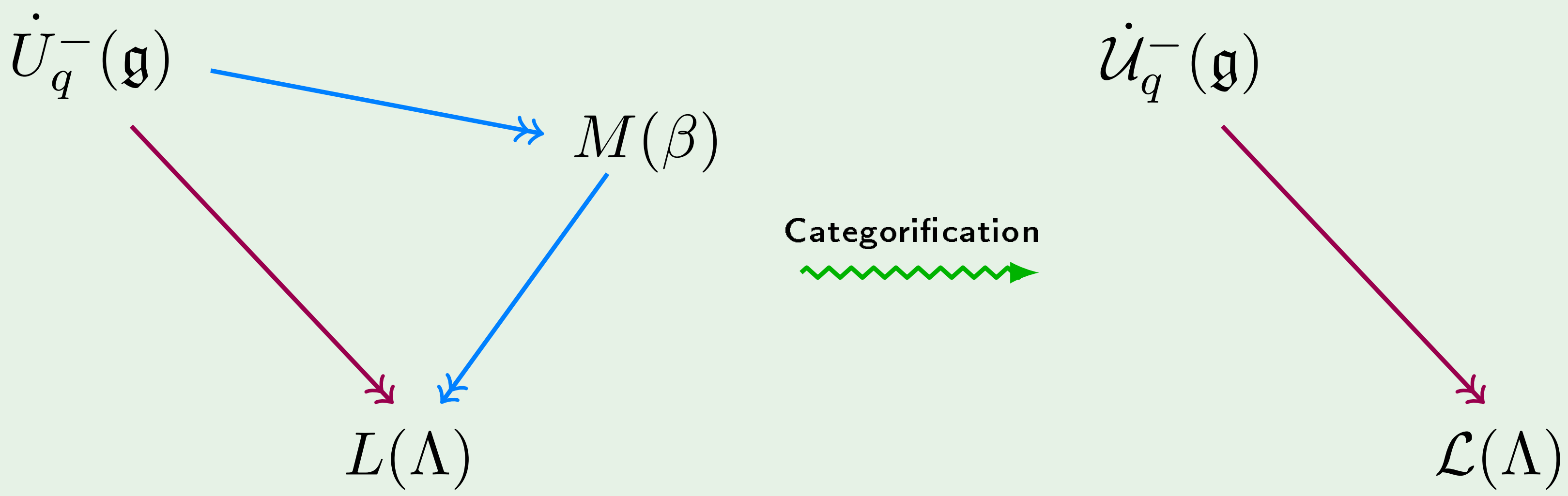

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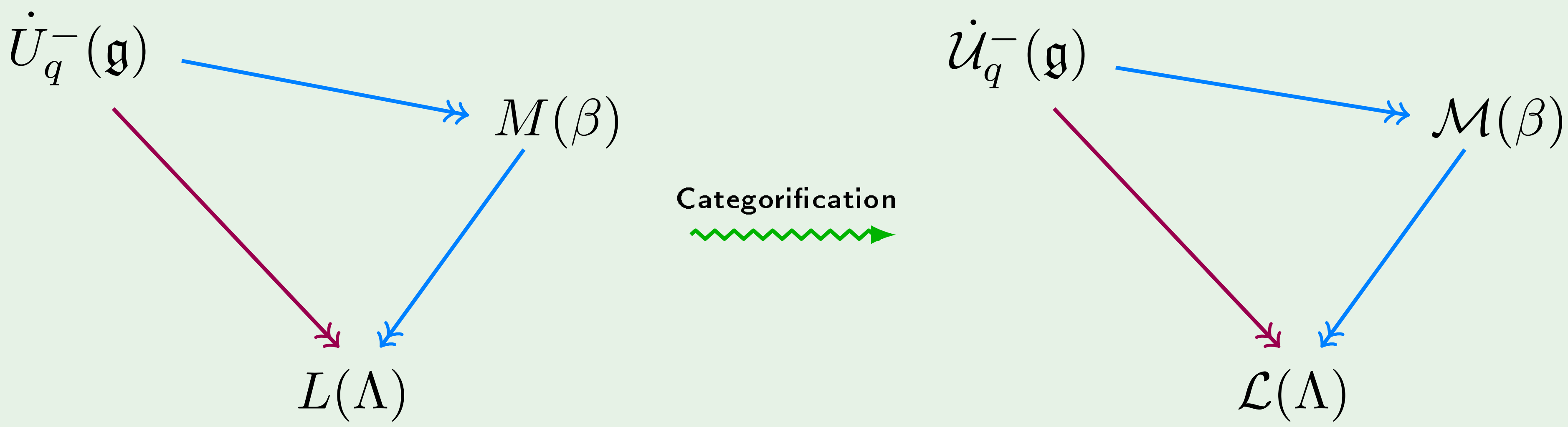

(cyclotomic) KLR algebras - I

Categorifications of $U_q^-(\mathfrak{g})$ and of $L(\Lambda)$ are given through KLR algebras.

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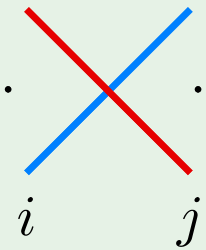
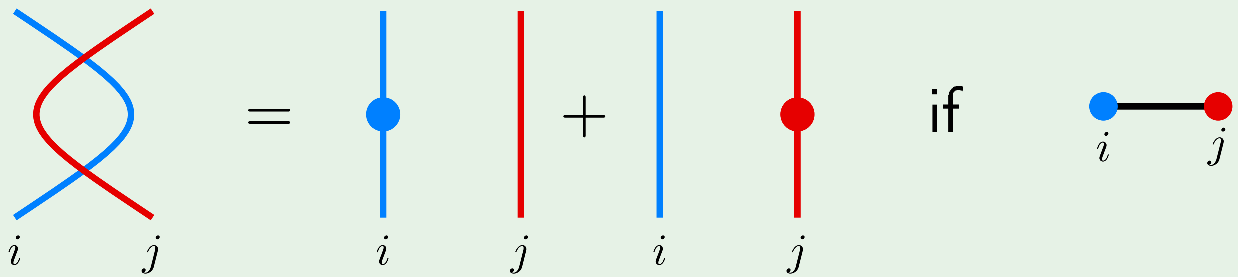
Fix $(\mathfrak{g}, I, \Lambda)$ and a ground ring $\Bbbk$.

KLR algebras can be defined by isotopy classes of diagrams / relations.

- Generators : $\cdots$  $\cdots$ and $\cdots$  $\cdots$ (for all $i, j \in I$).

- Relations (for example) :



(cyclotomic) KLR algebras - II

For $\nu = \sum_{i \in I} \nu_i \cdot i \in \mathbb{N}[I]$ let $R(\nu)$ be the algebra consisting of ν_i strands labelled i. Define

$$R = \bigoplus_{\nu \in \mathbb{N}[I]} R(\nu)$$

Theorem (Khovanov–Lauda, Rouquier '08)

$$K_0(R) \cong U_q^-(\mathfrak{g}).$$

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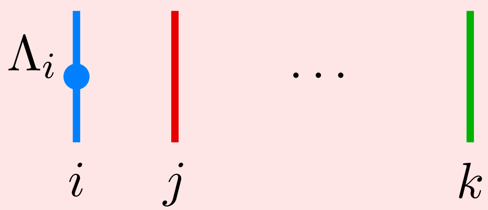
Let I^Λ be the 2-sided ideal generated by all pictures like



and put $$R^\Lambda = R/(I^\Lambda).$$

(cyclotomic) KLR algebras - III

Categorical $\mathfrak{g}$-action

We define

$$\mathrm{F}_i^{\Lambda}(\nu)\colon R^{\Lambda}(\nu)\text{-mod}_{\mathrm{g}} \to R^{\Lambda}(\nu+i)\text{-mod}_{\mathrm{g}}$$

as the functor of *induction* for the map that adds a strand labeled i at the right of a diagram from $R^{\Lambda}(\nu)$, and $\mathrm{E}_i^{\Lambda}(\nu)$ be its *right adjoint* (with an appropriated shift).

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These functors have very nice properties, for example they are *biadjoint* and the composites $\mathrm{E}_i^\Lambda\mathrm{F}_i^\Lambda(\nu)$ and $\mathrm{F}_i^\Lambda\mathrm{E}_i^\Lambda(\nu)$ satisfy a *direct sum decomposition* lifting the commutator relation.

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$$\mathrm{E}_i^\Lambda\mathrm{F}_i^\Lambda(\nu) \simeq \mathrm{F}_i^\Lambda\mathrm{E}_i^\Lambda(\nu) \oplus \bigoplus_{\ell=0}^{\nu_{i\pm1}-2\nu_i-1} \mathrm{Id}_\nu\{2\ell\} \qquad \text{if } \nu_{i\pm1} \geq 2\nu_i,$$

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$$F_k^\Lambda\mathrm{E}_j^\Lambda(\nu) \simeq \mathrm{E}_k^\Lambda\mathrm{F}_j^\Lambda(\nu) \qquad \text{for } j \neq k.$$

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The Categorification Theorem (Kang–Kashiwara, Webster,...)

$$K_0(R^\Lambda) \cong L(\Lambda) \qquad \text{(as } \mathfrak{g}\text{-modules)}$$

DG enhancements

We *cannot* categorify $M(\lambda)$ (nor $\Delta(\Lambda)$) from R!

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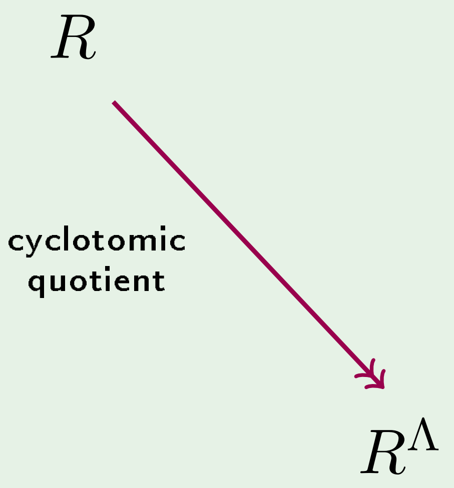

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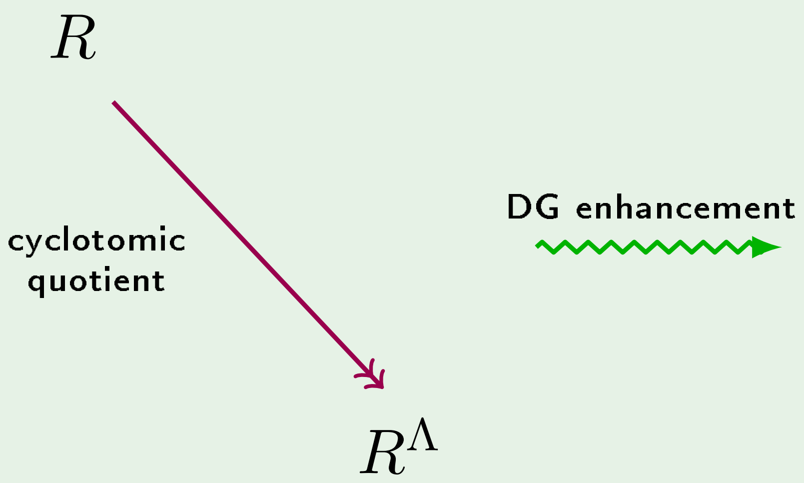

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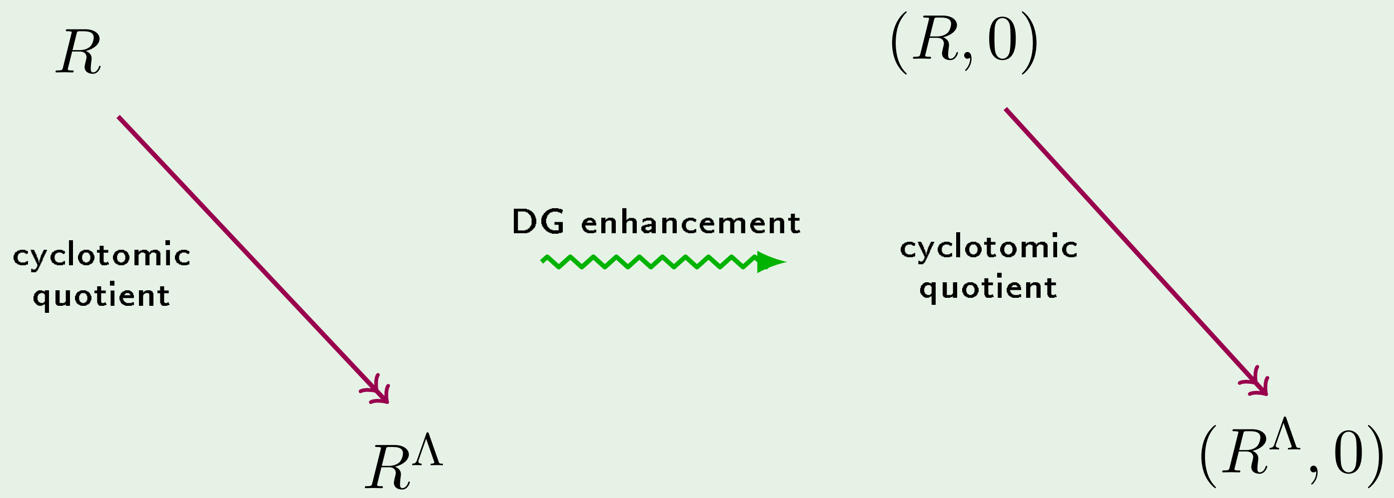

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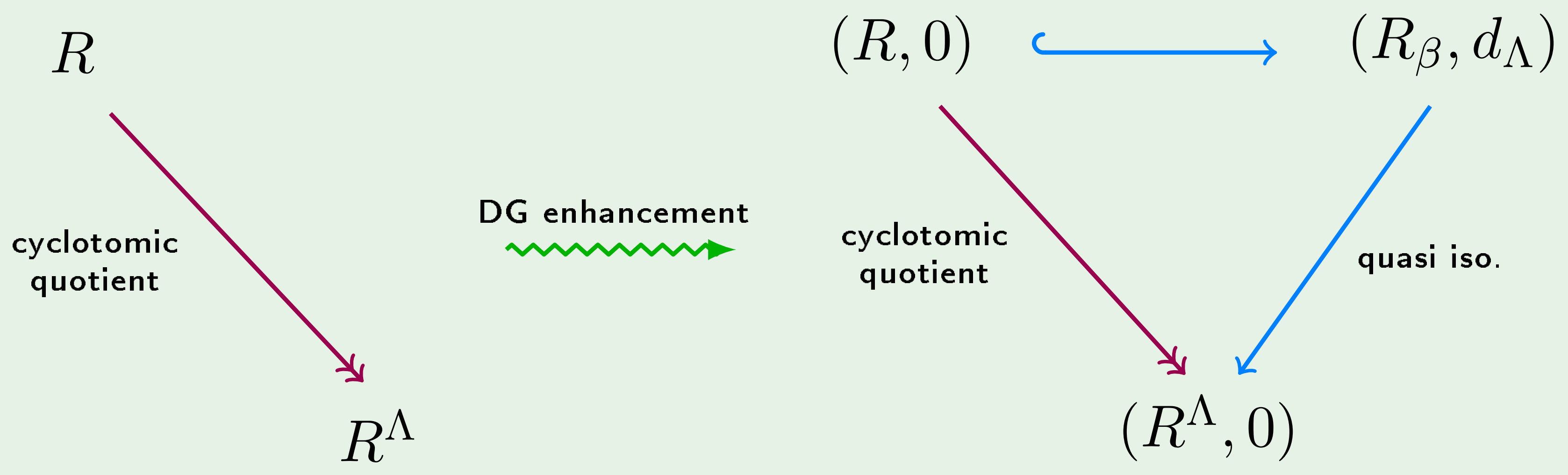
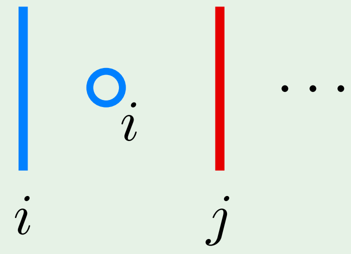


Extended KLR algebras - I

For each $i \in I$ add a new generator to R, a *floating dot* :



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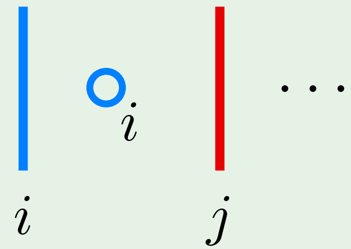

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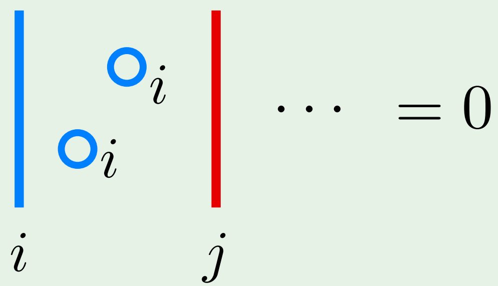
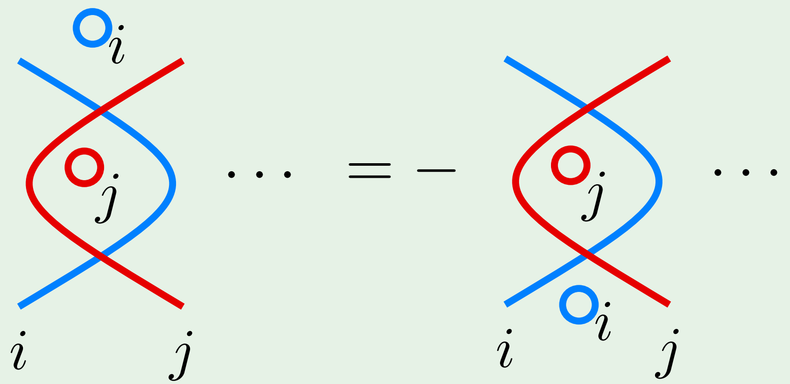
and impose two relations



and



This is a multigraded superalgebra where the KLR generators are *even* while floating dot are *odd*. Call this algebra R_β. This is a *minimal presentation*.

Extended KLR algebras - II

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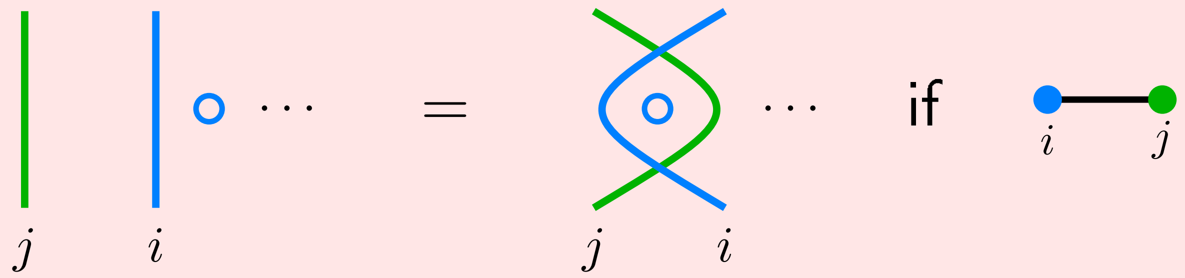
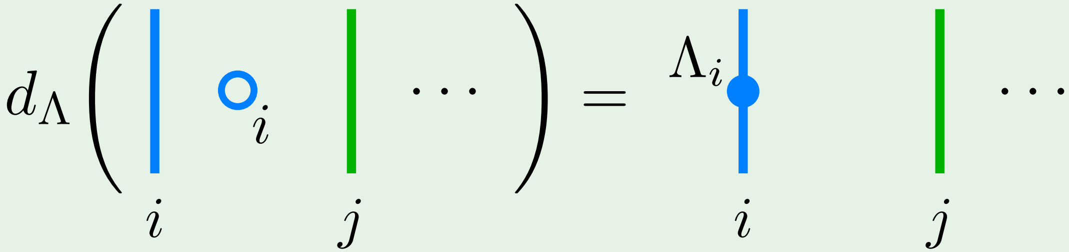
One can introduce more general floating dots, that can be placed in arbitray regions of the diagrams and get a presentation that is easier to handle in computations.

These generators satisfy some relations, for example,



Differentials : DG enhanced KLR algebras

For Λ an integral dominant weight for $\mathfrak{g}$ put



This defines a differential on R_β.

Proposition (Naisse–V. '17

The DG-algebras (R_β, d_Λ) and $(R^\Lambda, 0)$ are quasi-isomorphic.

- We say that (R_β, d_Λ) is a *DG-enhancement* of R^Λ.

Categorifying the half quantum group

Let $R_\beta(\nu)$ -mod$_{\mathrm{lf}}$ be the category of *left bounded*, *locally finite dimensional*, *left supermodules* over $R_\beta(\nu)$, with *degree zero morphisms*, and R_β-pmod$_{\mathrm{lf}}$ the (full) subcategory of projectives.

Categorifying the half quantum group

Let $R_\beta(\nu)\text{-mod}_{\mathrm{lf}}$ be the category of *left bounded, locally finite dimensional, left supermodules* over $R_\beta(\nu)$, with *degree zero morphisms*, and $R_\beta\text{-pmod}_{\mathrm{lf}}$ the (full) subcategory of projectives.

We can place diagrams aside each other to define an inclusion of algebras

$$R_\beta(\nu) \otimes R_{\beta-\mu}(\nu') \to R_\beta(\nu+\nu').$$

Theorem (Naisse–V. '17 :

There is an isomorphism of $\mathbb{Z}[q^{\pm 1}]$-*algebras*

$$K_0(\oplus_{\delta\in\mathbb{Z}[I]} R_{\beta+\delta}\text{-pmod}_{\mathrm{lf}}) \cong \dot{U}_q^-(\mathfrak{g}).$$

Categorifying Verma modules

Categorification of the weight spaces of $M(\beta)$

Put

$$\mathcal{M}(\beta) = R_\beta\text{-mod}_{\mathrm{lf}} = \bigoplus_{\nu \in \mathbb{N}[I]} R_\beta(\nu)\text{-mod}_{\mathrm{lf}}\,.$$

Define the functor

$$\mathrm{F}_i(\nu)\colon R_\beta(\nu)\text{-mod}_{\mathrm{lf}} \to R_\beta(\nu + i)\text{-mod}_{\mathrm{lf}}$$

as the functor of *induction* for the map that adds a strand colored i at the right of a diagram from $R_\beta(\nu)$, and denote $\mathrm{E}_i(\nu)$ its *right adjoint* (with some shift).

Categorifying Verma modules

There are several relations between these functors lifting the $\mathfrak{g}$-relations. For example,

Proposition (Naisse–V. '17)

There is a short exact sequence of functors

$$0 \to \mathrm{F}_i\mathrm{E}_i(\nu) \to \mathrm{E}_i\mathrm{F}_i(\nu) \to \mathrm{Q}(\nu)\langle q\mathsf{shift}_i, 1\rangle \oplus \Pi\mathrm{Q}(\nu)\langle -q\mathsf{shift}_i, -1\rangle \to 0$$

for all $i \in I$, and isomorphisms

$$\mathrm{F}_i\mathrm{E}_j(\nu) \simeq \mathrm{E}_j\mathrm{F}_i(\nu) \qquad \text{for } i \neq j.$$

Categorifying Verma modules

Put

$$\mathrm{F}_i = \bigoplus_{\nu \in \mathbb{N}[I]} \mathrm{F}_i(\nu) \qquad \text{and} \qquad \mathrm{E}_i = \bigoplus_{\nu \in \mathbb{N}[I]} \mathrm{E}_i(\nu).$$

The Categorification Theorem (Naisse–V. '17) :

• Functors $(\mathrm{F}_i, \mathrm{E}_i)$ are *exact* and form an *adjoint* pair.

• These functors induce an action of $U_q(\mathfrak{g})$ on the (topological) Grothendieck group of $\mathcal{M}(\beta)$.

With this action we have an isomorphism

$$K_0(\mathcal{M}(\beta)) \cong M(\beta),$$

of $U_q(\mathfrak{g})$-representations.

Categorification of $L(\Lambda)$

There is a SES

$$0 \to \left(\mathrm{F}_i\mathrm{E}_i(\nu), d_\Lambda\right) \to \left(\mathrm{E}_i\mathrm{F}_i(\nu), d_\Lambda\right) \to$$
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This induces a *LES in homology which splits*! Depending on the sign of $\nu_{i\pm1} - 2\nu_k$, the homology of the last term is concentrated in degree 0 or 1.

Corollary 1 :

$$E_i^\Lambda F_i^\Lambda(\nu) \simeq F_i^\Lambda E_i^\Lambda(\nu) \oplus \bigoplus_{\ell=0}^{\nu_{i\pm1}-2\nu_i-1} \mathrm{Id}_\nu\{2\ell\} \qquad \text{if } \nu_{i\pm1} \geq 2\nu_i,$$

$$F_i^\Lambda E_1^\Lambda(\nu) \simeq E_i^\Lambda F_i^\Lambda(\nu) \oplus \bigoplus_{\ell=0}^{2\nu_i-\nu_{i\pm1}-1} \mathrm{Id}_\nu\{2\ell\} \qquad \text{if } \nu_{i\pm1} \leq 2\nu_i,$$

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$$0 \to \big(\mathrm{F}_i\mathrm{E}_i(\nu), d_\Lambda\big) \to \big(\mathrm{E}_i\mathrm{F}_i(\nu), d_\Lambda\big) \to \big(\mathrm{Q}(\nu)\langle q\mathsf{shift}_i, 1\rangle \oplus \Pi\mathrm{Q}(\nu)\langle -q\mathsf{shift}_i, -1\rangle, d_\Lambda\big) \to 0$$

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Corollary 2 :

We have an *isomorphism*

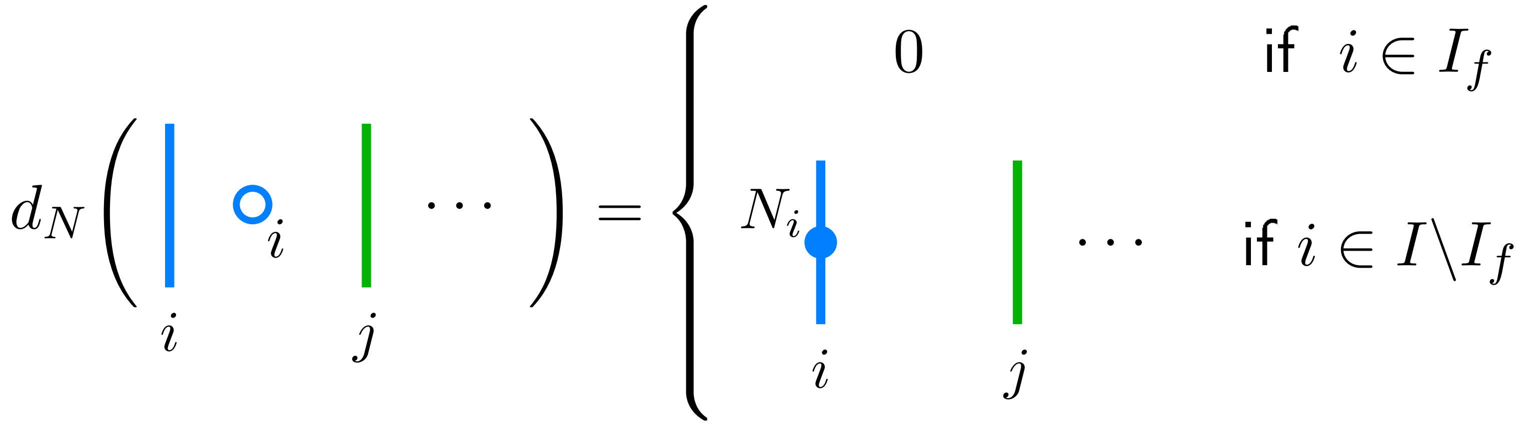
$$K_0\big(\mathcal{D}^c(R_\beta, d_\Lambda)\big) \cong L(\Lambda),$$

of $U_q(\mathfrak{g})$-representations.

Parabolic Verma modules

A subset $I_f \subseteq I$ defines a parabolic subalgebra $\mathfrak{p} \subseteq \mathfrak{g}$ and the construction above allow to category parabolic Verma modules as induced modules from the Levi factor $\mathfrak{l}$ of $\mathfrak{p}$ by redefining the differential d_Λ.

Let N be an integral dominant weight for $\mathfrak{l}$ and put



- This results in a categorification of the parabolic Verma module $M^{\mathfrak{p}}(V_N)$.

(Affine) Hecke algebras

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Let $\Bbbk$ be an algebraic closed field and fix $q \in \Bbbk^*$ and $d \in \mathbb{N}$ and let $H_d = H_d(q)$ be the

- *degenerate affine Hecke algebra* (over $\Bbbk$) if $q = 1$,

$$\{s_1, \ldots, s_{d-1}, X_1, \ldots, X_d\}/\text{relations},$$

  or the

- (non-degenerate) *affine Hecke algebra* (over $\Bbbk$) if $q \neq 1$,

$$\{T_1, \ldots, T_{d-1}, X_1^{\pm 1}, \ldots, X_d^{\pm 1}\}/\text{relations}.$$

Extended Hecke algebras

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Definition :

Define the superalgebra $\mathcal{H}_d$ by adding an *odd* variable θ to H_d and imposing the relations

$$\theta^2 = 0,$$

and

$$\begin{cases} \theta X_r = X_r \theta, \qquad s_i \theta = \theta s_i \quad \text{for} \quad i > 1 \\ \qquad s_1 \theta s_1 \theta + \theta s_1 \theta s_1 = 0 \end{cases} \qquad \text{if} \quad q = 1$$

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or

$$\begin{cases} \theta X_r = X_r\theta, \qquad T_i\theta = \theta T_i \quad \text{for} \quad i > 1 \\ T_1\theta T_1\theta + \theta T_1\theta T_1 = (q-1)\theta T_1\theta \end{cases} \qquad \text{if } q \neq 1$$

DG-enhanced cyclotomic Hecke algebras

Introduce a differential ∂_Λ on $\mathcal{H}_d$:

- ∂_Λ acts as zero on H_d while

$$\partial_\Lambda(\theta) = \begin{cases} \Pi_{i \in I}(X_1 - i)^{\Lambda_i} & \text{if } q = 1, \\ \Pi_{i \in I}(X_1 - q^i)^{\Lambda_i} & \text{if } q \neq 1, \end{cases} \quad + \quad \text{Leibniz rule}$$

Here, Λ is an integral dominant weight of Lie type A_∞ or $A_{n-1}^{(1)}$.

DG-enhanced cyclotomic Hecke algebras

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Here, Λ is an integral dominant weight of Lie type A_∞ or $A^{(1)}_{n-1}$.

Proposition (Maksimau–V. '18) :

The DG-algebras $(\mathcal{H}_d, \partial_\Lambda)$ and $(H^\Lambda_d, 0)$ are quasi-isomorphic.

Cyclotomic Hecke algebra : $$H^\Lambda_d = \begin{cases} \dfrac{H_d}{\Pi_{i\in I}(X_1 - i)^{\Lambda_i}} & \text{if } q = 1, \\ \dfrac{H_d}{\Pi_{i\in I}(X_1 - q^i)^{\Lambda_i}} & \text{if } q \neq 1. \end{cases}$$

The DG-enhanced BKR isomorphism

DG-enhanced cyclotomic KLR algebras

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- After a suitable completion, $\mathcal{H}_d$ gets a block decomposition where blocks are labelled by elements of $\mathbb{N}[I]$.
- The differentials ∂_Λ give rise to differentials on blocks.

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WIP (Maksimau–V.)

The DG-algebras $(\widehat{R}_\nu, d_\Lambda)$ and $(\widehat{\mathcal{H}}_d 1_\nu, \partial_{\Lambda,\nu})$ are isomorphic.

Thanks for your attention !