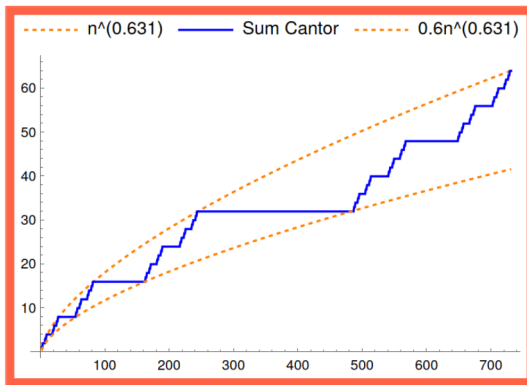


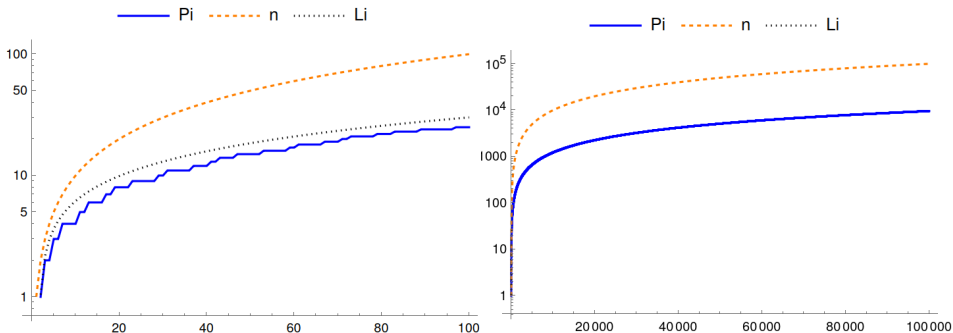
Analytic theory of monoidal categories

Or: Strategies to avoid counting



This is part 1

Let us not count!



- ▶ Prime number function $\pi(n) = \# \text{ primes } \leq n$
- ▶ Counting primes is very tricky as primes “pop up randomly”
- ▶ Question 1 What is the leading growth (of the number of primes)?
- ▶ Answer 1 There are roughly $c(n) \cdot n$ for sublinear correction term $c(n)$

Let us not count!

Seriously, counting is difficult!

Limite x	Nombre γ		Limite x	Nombre γ	
	par la formule.	par les Tables.		par la formule.	par les Tables.
10000	1230	1230	100000	9588	9592
20000	2268	2263	150000	13844	13849
30000	3252	3246	200000	17982	17984
40000	4205	4204	250000	22035	22045
50000	5136	5134	300000	26023	25998
60000	6049	6058	350000	29961	29977
70000	6949	6936	400000	33854	33861
80000	7838	7837	Actually, #primes < 1000 = 1229...		
90000	8717	8713			

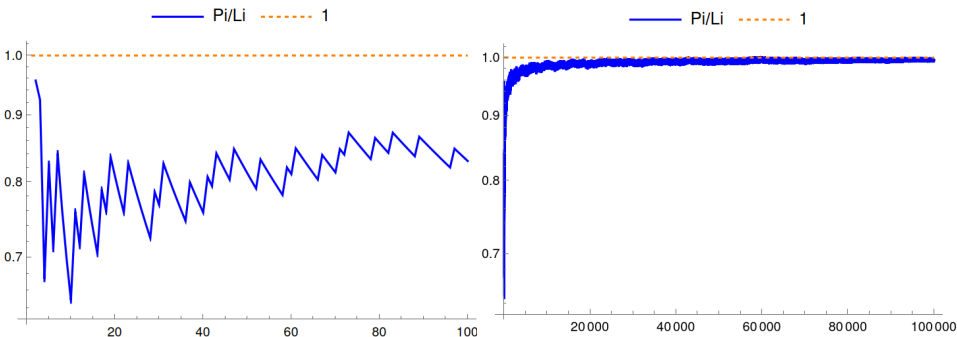
Legendre ~1808:
(for $n/(\ln n - 1.08366)$)

Gauss, Legendre and company counted primes up to $n = 400000$ and more

That took years (your iPhone can do that in seconds...humans have advanced!)

- ▶ **Question 1** What is the leading growth (of the number of primes)?
- ▶ **Answer 1** There are roughly $c(n) \cdot n$ for sublinear correction term $c(n)$

Let us not count!



- ▶ Asymptotically equal $f \sim g$ if $\lim_{n \rightarrow \infty} f(n)/g(n) \rightarrow 1$
- ▶ Logarithmic integral $\text{Li}(x) = \int_2^x 1/\ln(t) dt$
- ▶ Question 2 What is the growth (of the number of primes) asymptotically?
- ▶ Answer 2 We have $\pi(n) \sim n/\log(n) \sim \text{Li}(n)$

Riemann ~1859 calculates "the variance":

VII.

Ueber die Anzahl der Primzahlen unter einer
gegebenen Grösse.

(Monatsberichte der Berliner Akademie, November 1859.)

Durch Einsetzung dieser Werthe in den Ausdruck von $f(x)$ erhält man

$$f(x) = Li(x) - \sum^{\alpha} (Li(x^{\frac{1}{2} + \alpha i}) + Li(x^{\frac{1}{2} - \alpha i})) \\ + \int_x^{\infty} \frac{1}{x^2 - 1} \frac{dx}{x \log x} + \log \xi(0),$$

wenn in \sum^{α} für α sämtliche positiven (oder einen positiven reellen Theil enthaltenden) Wurzeln der Gleichung $\xi(\alpha) = 0$, ihrer Grösse nach geordnet, gesetzt werden. Es lässt sich, mit Hülfe einer genaueren Discussion der Function ξ , leicht zeigen, dass bei dieser Anordnung der Werth der Reihe

f is essentially the prime counting function π

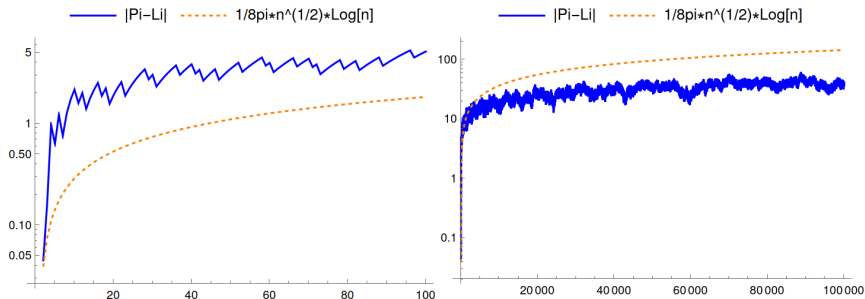
► Asy

► Log

► Que

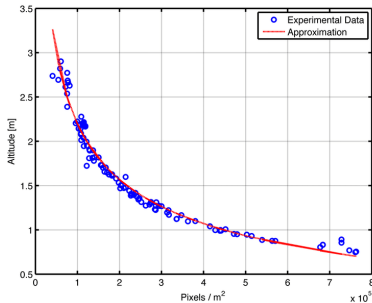
► Ans

Let us not count!



- ▶ Asymptotically equal does not imply that the difference is good
- ▶ $|f(n) - g(n)|$ is a measurement of how good the approximation is
- ▶ Question 3 What is variance from the expected value ($Li(n)$)?
- ▶ Conjectural answer 3 We have $|\pi(n) - Li(n)| \in O(n^{1/2} \log n)$ or $|\pi(n) - Li(n)| \leq \frac{1}{8\pi} n^{1/2} \log n$ (for $n \geq 2657$)

Let us not count!



- **Observation** Precise results are often out of reach but approximate answers are easy to get and beautiful
- These ideas are **ubiquitous** in discrete math, e.g. in number theory, combinatorics, graph theory, ...
- **Idea** Do the same in (rep theory + category theory) = monoidal categories



Course Outline

Today

(1) Motivating examples

After today

We focus on one specific problem:

Counting summands in tensor powers

Key examples are:

(2) Finite groups

(3) SL_2 + (4) dual problem

(5) General, including the Hecke category

► **Observation** Problems are easy to get a

► These ideas are in combinatorics, g

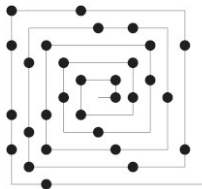
► **Idea** Do the same

approximate answers

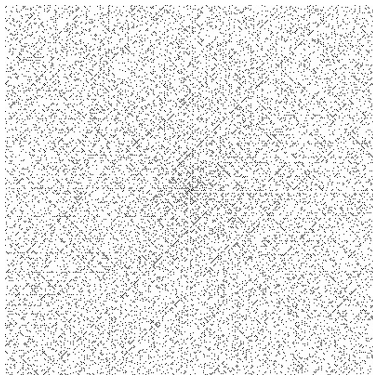
number theory,

monoidal categories

Not counting in graph theory

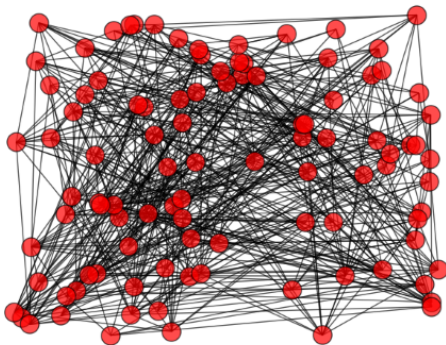


101	100	99	98	97	96	95	94	93	92	91
102	65	64	63	62	61	60	59	58	57	90
103	66	37	36	35	34	33	32	31	56	89
104	67	38	17	16	15	14	13	30	55	88
105	68	39	18	5	4	3	12	29	54	87
106	69	40	19	6	1	2	11	28	53	86
107	70	41	20	7	8	9	10	27	52	85
108	71	42	21	22	23	24	25	26	51	84
109	72	43	44	45	46	47	48	49	50	83
110	73	74	75	76	77	78	79	80	81	82
111	112	113	114	115	116	117	118	119	120	121



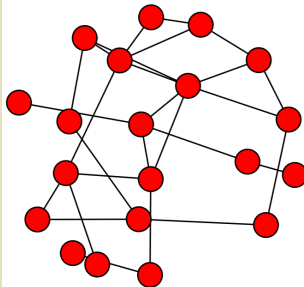
- ▶ Prime numbers appear essentially randomly
- ▶ Zooming out, they mostly look like noise
- ▶ However, also many patterns can be observed

Not counting in graph theory



-
- ▶ Random graphs = choose edges randomly = “average graphs”
 - ▶ Zooming out, they mostly look like noise
 - ▶ However, also many patterns can be observed

random graph with
20 nodes, 10% edge probability



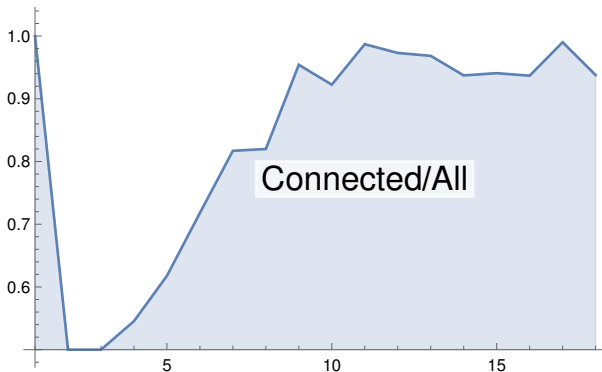
One studies random graphs $G_{n,p}$ for $n = |V| \gg 0$
with p =probability to connect vertices

► Random graphs "Asymptotically" many patterns arise

► Zooming out Example Almost all random graphs are connected

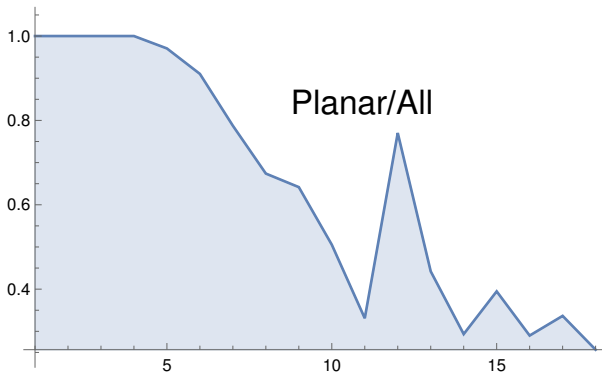
► However, also many patterns can be observed

Not counting in graph theory



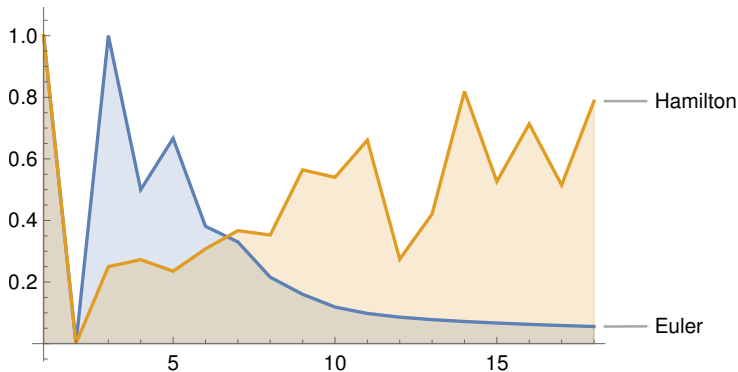
- **Fact** Most graphs have many edges
- Random graphs are **almost always** connected (=almost all graphs are connected)
- **Above** $\#$ connected graphs / $\#$ all graphs

Not counting in graph theory



- **Fact** Most graphs have many edges
- **Almost no** graph is planar
- **Above** $\frac{\# \text{ planar graphs}}{\# \text{ all graphs}}$

Not counting in graph theory

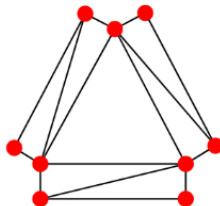


- **Fact** Most graphs have many edges
- **Almost all/no** graph is Hamiltonian/Eulerian
- **Above** $\#$ connected Hamil resp. Euler / $\#$ all graphs

Not counting in graph theory

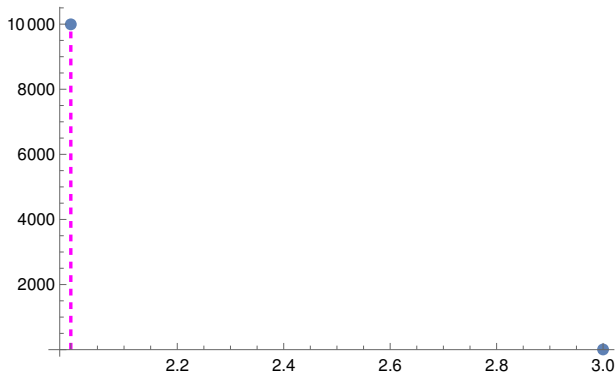
$ \text{Aut}(G) $	OEIS	counts of graphs with 1, 2, ... nodes
1	A003400	0, 0, 0, 0, 0, 8, 152, 3696, 135004, ...
2	A075095	0, 2, 2, 3, 11, 46, 354, 4431, 89004, ...
3		0, 0, 0, 0, 0, 0, 0, 4, ...
4	A075096	0, 0, 0, 2, 6, 36, 248, 2264, 31754, ...
6	A075097	0, 0, 2, 2, 2, 8, 38, 252, 3262, ...
8	A075098	0, 0, 0, 2, 4, 14, 74, 623, 7003, ...

*smallest cyclic group
graph
($n = 9$, $|\text{Aut}| = 3$)*



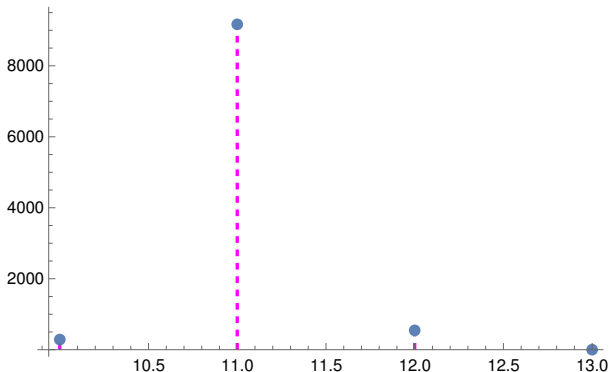
- ▶ $\text{Aut}(G)$ = group of automorphisms of a graph
- ▶ Graph automorphisms keep adjacency so random appearing edges are tricky
- ▶ Theorem Almost all graphs have trivial automorphism group

Not counting in graph theory



- ▶ Above The diameters of 10000 random coin flip graphs with 50 vertices
- ▶ Note the clustering
- ▶ Easy Almost all $G_{n,p}$ have $d(G_{n,p}) = 2$ (here and later $0 < p \leq 1$)

Not counting in graph theory

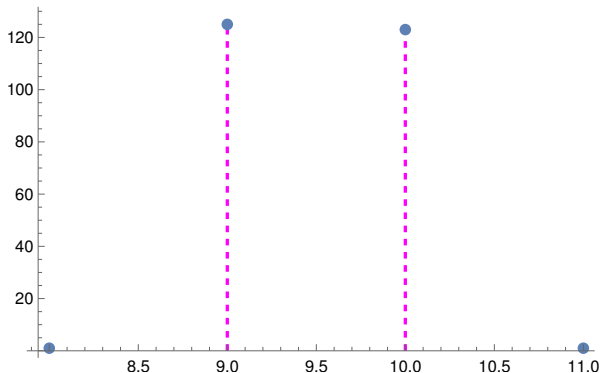


► Above Clique number cl of 10000 $G_{50,1/2}$

► There seems to be a peak at one value

► Indeed, the clique number satisfies $cl(G_{n,p}) \approx 2 \log_{1/p}(n)$

Not counting in graph theory



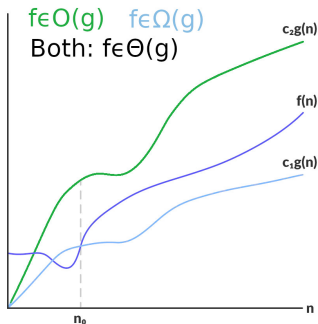
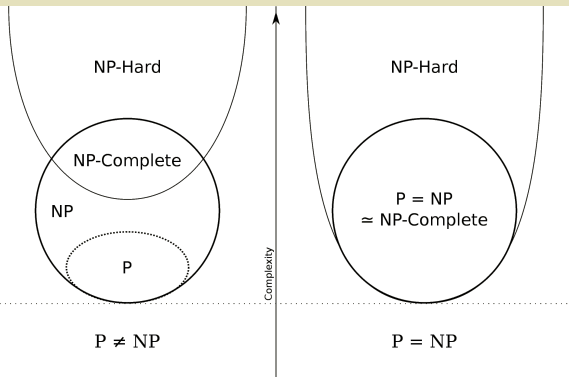
- Above χ of 250 $G_{50,1/2}$
- There seems to be a concentration around one or two values $\approx n/2 \log_2(n)$
- Easy The coloring number χ satisfies $\chi(G_{n,p}) \approx n/2 \log_{1/(1-p)}(n)$

Summary

“On average” and “asymptotic” answers might be nice
even if the precise results are very difficult

Example

Finding the chromatic number χ is NP-hard
The fastest known algorithms to find colorings are \approx in $O(n \cdot 2^n)$
But $\chi(G_{n,p}) \approx n/2 \log_{1/(1-p)} n$ is easy to get



Not counting in algebra

$gnu(n)$:

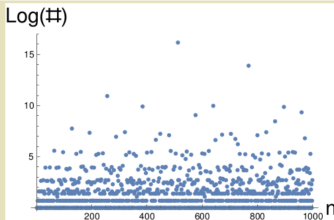
1	1	1	2	1	2	1	5	2	2	1	5
1	2	3	4	5	6	7	8	9	10	11	12
1	2	1	14	1	5	1	5	2	2	1	15
13	14	15	16	17	18	19	20	21	22	23	24
2	2	5	4	1	4	1	51	1	2	1	14
25	26	27	28	29	30	31	32	33	34	35	36
1	2	2	14	1	6	1	4	2	2	1	52
37	38	39	40	41	42	43	44	45	46	47	48
2	5	1	5	1	15	2	13	2	2	1	13
49	50	51	52	53	54	55	56	57	58	59	60
1	2	4	267	1	4	1	5	1	4	1	50
61	62	63	64	65	66	67	68	69	70	71	72
1	2	3	4	1	6	1	52	15	2	1	15
73	74	75	76	77	78	79	80	81	82	83	84
1	2	1	12	1	10	1	4	2	2	1	231
85	86	87	88	89	90	91	92	93	94	95	96
1	5	2	16	1	4	1	14	2	2	1	45
97	98	99	100	101	102	103	104	105	106	107	108
1	6	2	43	1	6	1	5	4	2	1	47
109	110	111	112	113	114	115	116	117	118	119	120
2	2	1	4	5	16	1	2328	2	4	1	10
121	122	123	124	125	126	127	128	129	130	131	132
1	2	5	15	1	4	1	11	1	2	1	197
133	134	135	136	137	138	139	140	141	142	143	144

- ▶ $gnu(n)$ = number of groups of order n
- ▶ Example $gnu(\text{prime}) = 1$, but getting other values is very hard
- ▶ Surprise One can prove nontrivial facts about $gnu(n)$

Finite groups are kind of random...

A000001 Number of groups of order n .
(Formerly M0098 N0035)

0, 1, 1, 1, 1, 2, 1, 2, 1, 5, 2, 2, 1, 5, 1, 2, 1, 14, 1, 5, 1, 5, 2, 2, 1, 15, 2, 2, 5, 4, 1, 4, 1, 51, 1, 2, 1, 14, 1, 2, 2, 14, 1, 6, 1, 4, 2, 2, 1, 52, 2, 5, 1, 5, 1, 15, 2, 13, 2, 2, 1, 13, 1, 2, 4, 267, 1, 4, 1, 5, 1, 4, 1, 50, 1, 2, 3, 4, 1, 6, 1, 52, 15, 2, 1, 15, 1, 2, 1, 12, 1, 10, 1,



- Pattern in a sea of randomness The 11758615 groups of order <1000 are swamped by the 10494213 of order 512
- However determining $gnu(n)$ precisely (even for prime powers) is very difficult
- So counting fails

Sims ~1964 For a prime we have $gnu(p^n) \sim p^{2n^3/27+O(n^{8/3})}$

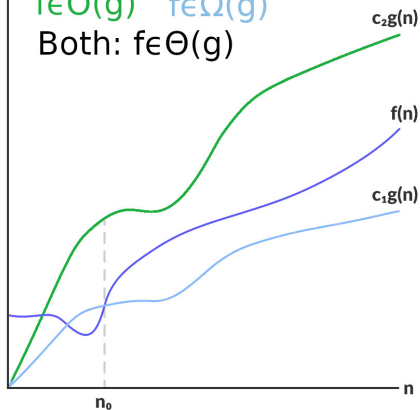
A000001 Num
(Form

0, 1, 1, 1, 2, 1
51, 1, 2, 1, 14,
2, 4, 267, 1, 4,

Reminder on capital O notation

$f \in O(g)$ $f \in \Omega(g)$

Both: $f \in \Theta(g)$



5, 4, 1, 4, 1,
2, 2, 1, 13, 1,
1, 12, 1, 10, 1,

- Pattern in a sequence is swamped by the growth of the sequence

- However, determining the growth of a sequence is very difficult

- So counting functions is very difficult

order < 1000 are

(powers) is very difficult

Theorem

Sims ~1964 For a prime we have $gnu(p^n) \sim p^{2n^3/27+O(n^{8/3})}$

Theorem

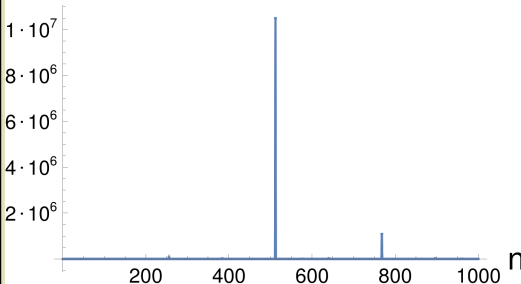
Kruse–Price ~1970, Blackburn–McLean ~2021

For a prime we have 'isoclasses of rings of size' $(p^n) \sim p^{4n^3/27+O(n^{8/3})}$

Folklore conjecture

Almost all groups are of order 2^n

$gnu(n)$



- Pattern in a swamped by

- However de

- So counting

order <1000 are

(s) is very difficult

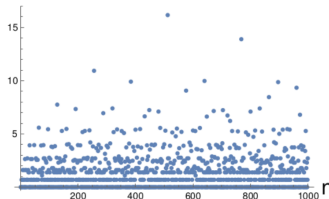
Finite groups are kind of random...

A000001 Number of groups of order n .

(Formerly M0098 N0035)

0, 1, 1, 1, 2, 1, 2, 1, 5, 2, 2, 1, 5, 1, 2, 1, 14, 1, 5, 1, 5, 2, 2, 1, 15, 2, 2, 5, 4, 1, 4, 1, 51, 1, 2, 1, 14, 1, 2, 2, 14, 1, 6, 1, 4, 2, 2, 1, 52, 2, 5, 1, 5, 1, 15, 2, 13, 2, 2, 1, 13, 1, 2, 4, 267, 1, 4, 1, 5, 1, 4, 1, 50, 1, 2, 3, 4, 1, 6, 1, 52, 15, 2, 1, 15, 1, 2, 1, 12, 1, 10, 1,

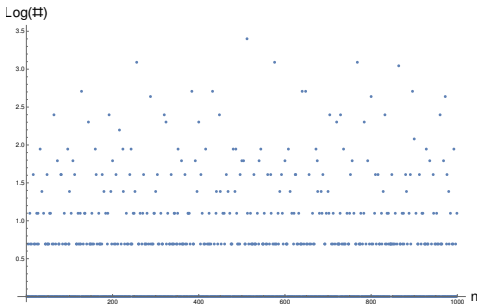
Log(#)



- Summary
- The gnu function $gnu(n)$ = number of different groups of size n
- Problem We know next to nothing about $gnu(n)$, but the 'growth' is fast

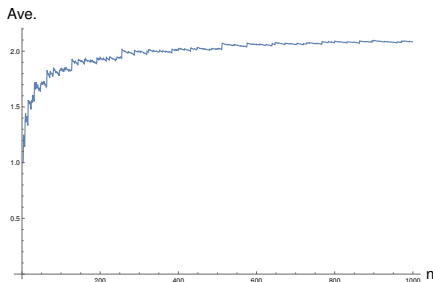
Not counting in algebra

A000688 Number of Abelian groups of order n ; number of factorizations of n into prime powers. 129
(Formerly M0064 N0020)
1, 1, 1, 2, 1, 1, 1, 3, 2, 1, 1, 2, 1, 1, 1, 5, 1, 2, 1, 2, 1, 1, 1, 3, 2, 1, 3, 2, 1, 1, 1, 7, 1,
1, 1, 4, 1, 1, 1, 5, 1, 1, 1, 2, 2, 1, 1, 5, 2, 2, 1, 2, 1, 3, 1, 1, 1, 1, 2, 1, 1, 2, 11, 1,
1, 1, 2, 1, 1, 1, 6, 1, 1, 2, 2, 1, 1, 1, 5, 5, 1, 1, 2, 1, 1, 1, 3, 1, 2, 1, 2, 1, 1, 1, 7, 1, 2,
2, 4, 1, 1, 1, 3, 1, 1, 1 ([list](#); [graph](#); [refs](#); [listen](#); [history](#); [text](#); [internal format](#))



- ▶ Abelian groups of order n = symmetries with n commuting operations
- ▶ The agnu function $agnu(n)$ = number of different abelian groups of size n
- ▶ Task Describe $agnu(n)$

Not counting in algebra



- The average number of abelian groups of a given order is

$$\prod_{j \geq 2} \zeta(j) \approx 2.29485659$$

- ζ is the (Riemann) zeta function

- Average is in the sense of arithmetic mean : $\frac{1}{n} \sum_{k=1}^n agnu(k)$

Ave.

Theorem

Knopfmacher ~1972 The average number of semisimple rings is

$$\prod_{rm^2 > 1} \zeta(rm^2) \approx 2.49961611$$

Semisimple = matrix rings

$$\begin{array}{c}
 \begin{matrix} 1 & 2 & \dots & n \end{matrix} \\
 \begin{matrix} 1 \\ 2 \\ 3 \\ \vdots \\ m \end{matrix} \left[\begin{array}{cccc} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ a_{31} & a_{32} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{array} \right]
 \end{array}$$

► The a

► ζ is the (Riemann) zeta function

► Average is in the sense of arithmetic mean : $\frac{1}{n} \sum_{k=1}^n agnu(k)$

Not counting in representation theory

D_4 :

Class		1	2	3	4	5
Size		1	1	2	2	2
Order		1	2	2	2	4

p	=	2	1	1	1	1

X.1	+	1	1	1	1	1
X.2	+	1	1	-1	1	-1
X.3	+	1	1	1	-1	-1
X.4	+	1	1	-1	-1	1
X.5	+	2	-2	0	0	0

, Q_8 :

Class		1	2	3	4	5
Size		1	1	2	2	2
Order		1	2	4	4	4

p	=	2	1	1	2	2

X.1	+	1	1	1	1	1
X.2	+	1	1	-1	1	-1
X.3	+	1	1	1	-1	-1
X.4	+	1	1	-1	-1	1
X.5	-	2	-2	0	0	0

- ▶ Characters do **not** determine finite groups
- ▶ **First example** The dihedral group D_4 (eight elements) and Q_8 (quaternions)
- ▶ **Question** How good is the character table?

Not counting in representation theory

Class | 1 2 3 4 5

Class | 1 2 3 4 5

Some patterns

Frobenius? ~1900 Abelian groups are determined by their characters
(Even better: two groups with the same characters have isomorphic abelianizations)

Oyama ~1964 Symmetric groups are determined by their characters

Many people ~1960++ Nonabelian simple groups are determined by their characters

X.3 + 1 1 1 -1 -1
X.4
X.5

Problem Brauer ~1963

Classify the groups that are determined by their characters

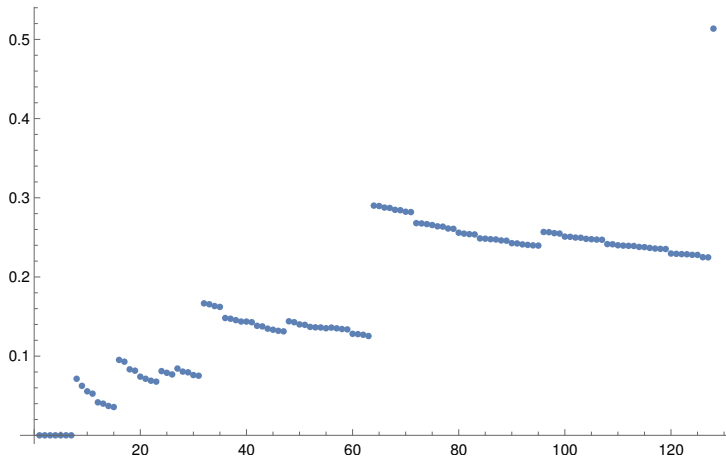
► Character

This is probably out of reach

► **First example** The dihedral group D_4 (eight elements) and Q_8 (quaternions)

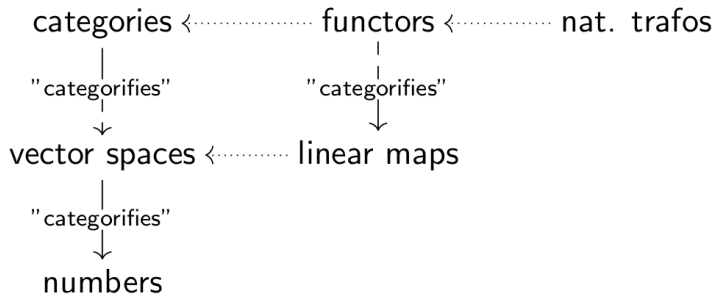
► **Question** How good is the character table?

Not counting in representation theory



- Folklore conjecture Almost all group are not determined by characters
- Above % of groups $\leq n$ not determined by their characters

Not counting in representation theory



- ▶ 'Categorification' Are groups determined by their representation categories?
- ▶ In formulas (complex coefficients)

$$(\mathbf{Rep}(G) \cong_{\otimes} \mathbf{Rep}(H)) \Rightarrow (G \cong H)?$$

Such groups are called categorically rigid

Not counting in representation theory

Some patterns (all **Etingof–Gelaki** ~2000)

All groups that are determined by their characters are categorically rigid

All groups of order $\equiv 1$ or 2 or $3 \pmod{4}$ are categorically rigid

All groups of order < 64 are categorically rigid

Categorically rigid is independent of the field (as long as $\mathbb{K} = \bar{\mathbb{K}}$)

► ‘Categorical’ **Theorem Etingof–Gelaki, Davydov, Izumi–Kosaki** ~2000 categories?

► In form One can classify the groups that are determined by their representation categories

Categorification helps!

Such groups are called categorically rigid

Not counting in representation theory

	n	$n \log_2 n$	n^2	n^3	1.5^n	2^n	$n!$
$n = 10$	< 1 sec	< 1 sec	< 1 sec	< 1 sec	< 1 sec	< 1 sec	4 sec
$n = 30$	< 1 sec	< 1 sec	< 1 sec	< 1 sec	< 1 sec	18 min	10^{25} years
$n = 50$	< 1 sec	< 1 sec	< 1 sec	< 1 sec	11 min	36 years	very long
$n = 100$	< 1 sec	< 1 sec	< 1 sec	1 sec	12,892 years	10^{17} years	very long
$n = 1,000$	< 1 sec	< 1 sec	1 sec	18 min	very long	very long	very long
$n = 10,000$	< 1 sec	< 1 sec	2 min	12 days	very long	very long	very long
$n = 100,000$	< 1 sec	2 sec	3 hours	32 years	very long	very long	very long
$n = 1,000,000$	1 sec	20 sec	12 days	31,710 years	very long	very long	very long

Polynomial time		Exponential Time	
n	- Linear Search	2^n	- 0/1 knapsack
$\log n$	- Binary Search	2^n	- Travelling SP
$n * n$	- Insertion Sort	2^n	- Sum of Subsets
$n * \log n$	- Merge Sort	2^n	- Graph Coloring
$n * n * n$	- Matrix Multiplication	2^n	- Hamilton Cycle

- **Problem** Checking this is very difficult
- **Compare** Computing character tables is $O(\text{poly in } \#G)$

Theorem Deligne–Milne ~1982

If you include symmetry (the 'braiding'), then
all groups are determined by $\mathbf{Rep}(G)$

$$\begin{array}{c} W_{R_i} \quad W_{R_j} \quad W_{R_k} \\ \text{[Three vertical lines]} \end{array} = \sum_{R_k} N_{R_i, R_j}^{R_k}$$

$$\begin{array}{c} W_{R_i} \quad W_{R_j} \\ \text{[Y-junction]} \\ W_{R_k} \quad W_{R_l} \end{array} = F_{R_i R_j R_k}^{R_l} \begin{array}{c} W_{R_i} \quad W_{R_j} \\ \text{[Horizontal line]} \\ W_{R_k} \quad W_{R_l} \end{array}$$

$$\begin{array}{c} W_{R_i} \quad W_{R_j} \\ \text{[Y-junction]} \\ \text{[Loop]} \\ W_{R_k} \end{array} = R_{R_i R_j}^{R_k} \begin{array}{c} W_{R_i} \quad W_{R_j} \\ \text{[Y-junction]} \\ W_{R_k} \end{array}$$

 $n!$

4 sec

 10^{25} years

very long

very long

very long

very long

very long

very long

$n = 10$
 $n = 30$
 $n = 50$
 $n = 100$
 $n = 1,000$
 $n = 10,000$
 $n = 100,000$
 $n = 1,000,000$

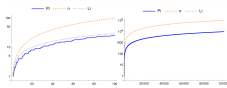
n
 \log
 n^*
 n^*
 n^*

► Problem Check

► Compare Computing character tables is $O(\text{poly in } \#G)$

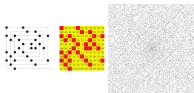
This is however even more difficult to check

Let us not count!



- Prime number function $\pi(n) = \#\text{primes} \leq n$
- Counting primes is **very tricky** as primes "pop up randomly"
- **Question 1** What is the leading growth (of the number of primes)?
- **Answer 1** There are roughly $c(n) \cdot n$ for sublinear correction term $c(n)$

Not counting in graph theory



- **Prime numbers** appear essentially randomly
- Zooming out, they mostly look like **points**
- However, also **many patterns** can be observed

Not counting in algebra



- The average number of abelian groups of a given order is $\prod_{p|n} (1 + \frac{1}{p}) \approx 2.29489659$
- ζ is the (Riemann) **zeta function**
- Average is in the sense of **arithmetic mean** $\frac{1}{n} \sum_{d|n} \mu(d) \cdot k(d)$

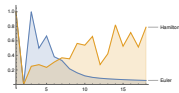
Let us not count!

Seriously, counting is difficult!

Legendre	Gauss	Legendre and Gauss
1792	1792	1792
1793	1793	1793
1794	1794	1794
1795	1795	1795
1796	1796	1796
1797	1797	1797
1798	1798	1798
1799	1799	1799
1800	1800	1800
1801	1801	1801
1802	1802	1802
1803	1803	1803
1804	1804	1804
1805	1805	1805
1806	1806	1806
1807	1807	1807
1808	1808	1808
1809	1809	1809
1810	1810	1810
1811	1811	1811
1812	1812	1812
1813	1813	1813
1814	1814	1814
1815	1815	1815
1816	1816	1816
1817	1817	1817
1818	1818	1818
1819	1819	1819
1820	1820	1820
1821	1821	1821
1822	1822	1822
1823	1823	1823
1824	1824	1824
1825	1825	1825
1826	1826	1826
1827	1827	1827
1828	1828	1828
1829	1829	1829
1830	1830	1830
1831	1831	1831
1832	1832	1832
1833	1833	1833
1834	1834	1834
1835	1835	1835
1836	1836	1836
1837	1837	1837
1838	1838	1838
1839	1839	1839
1840	1840	1840
1841	1841	1841
1842	1842	1842
1843	1843	1843
1844	1844	1844
1845	1845	1845
1846	1846	1846
1847	1847	1847
1848	1848	1848
1849	1849	1849
1850	1850	1850
1851	1851	1851
1852	1852	1852
1853	1853	1853
1854	1854	1854
1855	1855	1855
1856	1856	1856
1857	1857	1857
1858	1858	1858
1859	1859	1859
1860	1860	1860
1861	1861	1861
1862	1862	1862
1863	1863	1863
1864	1864	1864
1865	1865	1865
1866	1866	1866
1867	1867	1867
1868	1868	1868
1869	1869	1869
1870	1870	1870
1871	1871	1871
1872	1872	1872
1873	1873	1873
1874	1874	1874
1875	1875	1875
1876	1876	1876
1877	1877	1877
1878	1878	1878
1879	1879	1879
1880	1880	1880
1881	1881	1881
1882	1882	1882
1883	1883	1883
1884	1884	1884
1885	1885	1885
1886	1886	1886
1887	1887	1887
1888	1888	1888
1889	1889	1889
1890	1890	1890
1891	1891	1891
1892	1892	1892
1893	1893	1893
1894	1894	1894
1895	1895	1895
1896	1896	1896
1897	1897	1897
1898	1898	1898
1899	1899	1899
1900	1900	1900
1901	1901	1901
1902	1902	1902
1903	1903	1903
1904	1904	1904
1905	1905	1905
1906	1906	1906
1907	1907	1907
1908	1908	1908
1909	1909	1909
1910	1910	1910
1911	1911	1911
1912	1912	1912
1913	1913	1913
1914	1914	1914
1915	1915	1915
1916	1916	1916
1917	1917	1917
1918	1918	1918
1919	1919	1919
1920	1920	1920
1921	1921	1921
1922	1922	1922
1923	1923	1923
1924	1924	1924
1925	1925	1925
1926	1926	1926
1927	1927	1927
1928	1928	1928
1929	1929	1929
1930	1930	1930
1931	1931	1931
1932	1932	1932
1933	1933	1933
1934	1934	1934
1935	1935	1935
1936	1936	1936
1937	1937	1937
1938	1938	1938
1939	1939	1939
1940	1940	1940
1941	1941	1941
1942	1942	1942
1943	1943	1943
1944	1944	1944
1945	1945	1945
1946	1946	1946
1947	1947	1947
1948	1948	1948
1949	1949	1949
1950	1950	1950
1951	1951	1951
1952	1952	1952
1953	1953	1953
1954	1954	1954
1955	1955	1955
1956	1956	1956
1957	1957	1957
1958	1958	1958
1959	1959	1959
1960	1960	1960
1961	1961	1961
1962	1962	1962
1963	1963	1963
1964	1964	1964
1965	1965	1965
1966	1966	1966
1967	1967	1967
1968	1968	1968
1969	1969	1969
1970	1970	1970
1971	1971	1971
1972	1972	1972
1973	1973	1973
1974	1974	1974
1975	1975	1975
1976	1976	1976
1977	1977	1977
1978	1978	1978
1979	1979	1979
1980	1980	1980
1981	1981	1981
1982	1982	1982
1983	1983	1983
1984	1984	1984
1985	1985	1985
1986	1986	1986
1987	1987	1987
1988	1988	1988
1989	1989	1989
1990	1990	1990
1991	1991	1991
1992	1992	1992
1993	1993	1993
1994	1994	1994
1995	1995	1995
1996	1996	1996
1997	1997	1997
1998	1998	1998
1999	1999	1999
2000	2000	2000

- **Question 2** What is the leading growth (of the number of primes)?
- **Answer 2** There are roughly $c(n) \cdot n$ for sublinear correction term $c(n)$

Not counting in graph theory



- **Fact** Most graphs have many edges
- **Almost all/no** graph is Hamiltonian/Eulerian
- **Above** π connected Hamilt resp. Euler \neq all graphs

Not counting in representation theory

Class	Size	Order
1	1	1
2	2	2
3	3	3
4	4	4
5	5	5
6	6	6
7	7	7
8	8	8
9	9	9
10	10	10
11	11	11
12	12	12
13	13	13
14	14	14
15	15	15
16	16	16
17	17	17
18	18	18
19	19	19
20	20	20
21	21	21
22	22	22
23	23	23
24	24	24
25	25	25
26	26	26
27	27	27
28	28	28
29	29	29
30	30	30
31	31	31
32	32	32
33	33	33
34	34	34
35	35	35
36	36	36
37	37	37
38	38	38
39	39	39
40	40	40
41	41	41
42	42	42
43	43	43
44	44	44
45	45	45
46	46	46
47	47	47
48	48	48
49	49	49
50	50	50
51	51	51
52	52	52
53	53	53
54	54	54
55	55	55
56	56	56
57	57	57
58	58	58
59	59	59
60	60	60
61	61	61
62	62	62
63	63	63
64	64	64
65	65	65
66	66	66
67	67	67
68	68	68
69	69	69
70	70	70
71	71	71
72	72	72
73	73	73
74	74	74
75	75	75
76	76	76
77	77	77
78	78	78
79	79	79
80	80	80
81	81	81
82	82	82
83	83	83
84	84	84
85	85	85
86	86	86
87	87	87
88	88	88
89	89	89
90	90	90
91	91	91
92	92	92
93	93	93
94	94	94
95	95	95
96	96	96
97	97	97
98	98	98
99	99	99
100	100	100

- **Character table** = prototypical rep theory
- **Example** The character tables of S_3 and S_4 created with **Magma**
- **What do we use?** Rows = simple characters, columns = conjugacy classes, size = number of elements, order = order of elements

- Analysis theory of bounded categories On Strategies to avoid counting July 2004 2 / 6

- | | | | |
|--------------------------------------|---------------------------------|-----------|-------|
| Analysis theory of nested categories | On Strategies to count counting | July 2008 | 1 / 6 |
|--------------------------------------|---------------------------------|-----------|-------|

- $\Pi_{Q23}(j) \approx 2.29485659$

- | | | |
|---|---------------------------------|-----------------|
| Analysis theory of essential categories | On strategies to avoid counting | July 2018 8 / 8 |
|---|---------------------------------|-----------------|

Seriously, counting is difficult!					
Number of			Number of		
Line(s) n	per 1 female (see Table)		Line(s) n	per 1 female (see Table)	
13000	1000	1350	100000	2500	5000
14000	2500	2500	110000	1500	1500
15000	5000	5000	120000	1000	1000
16000	4000	4000	130000	1000	1000
17000	1000	1000	140000	1000	1000
18000	6000	6000	150000	2000	2000
19000	6000	6000	160000	2000	2000
20000	1000	1000	170000	2000	2000
21000	8000	8000	180000	2000	2000
22000	8000	8000	190000	2000	2000
23000	8000	8000	200000	2000	2000
24000	8000	8000	210000	2000	2000
25000	8000	8000	220000	2000	2000
26000	8000	8000	230000	2000	2000
27000	8000	8000	240000	2000	2000
28000	8000	8000	250000	2000	2000
29000	8000	8000	260000	2000	2000
30000	8000	8000	270000	2000	2000
31000	8000	8000	280000	2000	2000
32000	8000	8000	290000	2000	2000
33000	8000	8000	300000	2000	2000
34000	8000	8000	310000	2000	2000
35000	8000	8000	320000	2000	2000
36000	8000	8000	330000	2000	2000
37000	8000	8000	340000	2000	2000
38000	8000	8000	350000	2000	2000
39000	8000	8000	360000	2000	2000
40000	8000	8000	370000	2000	2000
41000	8000	8000	380000	2000	2000
42000	8000	8000	390000	2000	2000
43000	8000	8000	400000	2000	2000
44000	8000	8000	410000	2000	2000
45000	8000	8000	420000	2000	2000
46000	8000	8000	430000	2000	2000
47000	8000	8000	440000	2000	2000
48000	8000	8000	450000	2000	2000
49000	8000	8000	460000	2000	2000
50000	8000	8000	470000	2000	2000
51000	8000	8000	480000	2000	2000
52000	8000	8000	490000	2000	2000
53000	8000	8000	500000	2000	2000
54000	8000	8000	510000	2000	2000
55000	8000	8000	520000	2000	2000
56000	8000	8000	530000	2000	2000
57000	8000	8000	540000	2000	2000
58000	8000	8000	550000	2000	2000
59000	8000	8000	560000	2000	2000
60000	8000	8000	570000	2000	2000
61000	8000	8000	580000	2000	2000
62000	8000	8000	590000	2000	2000
63000	8000	8000	600000	2000	2000
64000	8000	8000	610000	2000	2000
65000	8000	8000	620000	2000	2000
66000	8000	8000	630000	2000	2000
67000	8000	8000	640000	2000	2000
68000	8000	8000	650000	2000	2000
69000	8000	8000	660000	2000	2000
70000	8000	8000	670000	2000	2000
71000	8000	8000	680000	2000	2000
72000	8000	8000	690000	2000	2000
73000	8000	8000	700000	2000	200

- $\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$

n	Euler Probability	Hamilton Probability
1	1.00	0.00
2	1.00	0.00
3	0.60	0.10
4	0.65	0.20
5	0.40	0.25
6	0.35	0.30
7	0.30	0.40
8	0.25	0.45
9	0.20	0.50
10	0.15	0.55
11	0.10	0.60
12	0.08	0.40
13	0.07	0.45
14	0.06	0.80
15	0.05	0.55
16	0.04	0.65
17	0.03	0.50
18	0.02	0.60
19	0.01	0.70
20	0.01	0.80

- Analytic theory of modular congruences 60 Strategies to avoid counting July 2018 7 / 4

Class	1	2	3	4	5
Size	1	3	2	6	4
Order	1	2	3	2	4

Class	1	2	3	4	5
Size	1	3	6	4	5
Order	1	2	2	3	4

Class	1	2	3	4	5
Size	1	3	1	1	4
Order	1	2	1	3	4

Class	1	2	3	4	5
Size	1	3	1	1	1
Order	1	2	2	3	4

Class	1	2	3	4	5
Size	1	3	1	1	1
Order	1	2	2	3	4

Class	1	2	3	4	5
Size	1	3	1	1	1
Order	1	2	2	3	4

Class	1	2	3	4	5
Size	1	3	1	1	1
Order	1	2	2	3	4

Class	1	2	3	4	5
Size	1	3	1	1	1
Order	1	2	2	3	4

Class	1	2	3	4	5
Size	1	3	1	1	1
Order	1	2	2	3	4

Class	1	2	3	4	5
Size	1	3	1	1	1
Order	1	2	2	3	4

Class	1	2	3	4	5
Size	1	3	1	1	1
Order	1	2	2	3	4

Class	1	2	3	4	5
Size	1	3	1	1	1
Order	1	2	2	3	4

Class	1	2	3	4	5
Size	1	3	1	1	1
Order	1	2	2	3	4

Class	1	2	3	4	5
Size	1	3	1	1	1
Order	1	2	2	3	4

Class	1	2	3	4	5
Size	1	3	1	1	1
Order	1	2	2	3	4

Class	1	2	3	4	5
Size	1	3	1	1	1
Order	1	2	2	3	4

Class	1	2	3	4	5
Size	1	3	1	1	1
Order	1	2	2	3	4

Class	1	2	3	4	5
Size	1	3	1	1	1
Order	1	2	2	3	4

Class	1	2	3	4	5
Size	1	3	1	1	1
Order	1	2	2	3	4

Class	1	2	3	4	5
Size	1	3	1	1	1
Order	1	2	2	3	4

Class	1	2	3	4	5
Size	1	3	1	1	1
Order	1	2	2	3	4

Class	1	2	3	4	5
Size	1	3	1	1	1
Order	1	2	2	3	4

Class	1	2	3	4	5
Size	1	3	1	1	1
Order	1	2	2	3	4

Class	1	2	3	4	5
Size	1	3	1		

- | | | | |
|---|---------------------------------|-----------|-------|
| Analysis theory of oscillatory singular | Dr. Strategia is solid counting | July 2020 | 5 / 4 |
|---|---------------------------------|-----------|-------|

- **Conjectural lower bound:** The same $|a(n) - \text{Li}(n)| \in O(n^{-1/2} \log n)$ ($|a(n) - f(n)| \leq \frac{1}{2} n^{1/2} \log n$ (for $n \geq 2657$)).

[illegible]

- Analytic theory of resonant coupling On Strategies to avoid coupling July 2014 8 / 14

- | | | | |
|--------------------------------------|---------------------------------|-----------|-------|
| Analysis theory of nested categories | On Strategies to avoid counting | July 2014 | 5 / 4 |
|--------------------------------------|---------------------------------|-----------|-------|

Thanks for your attention!