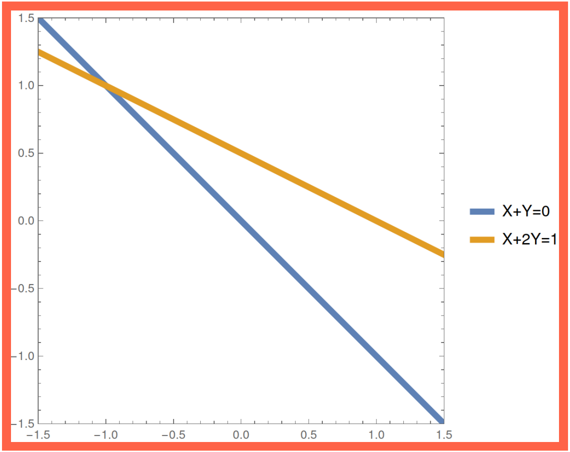
What is...tropical geometry - part 9?

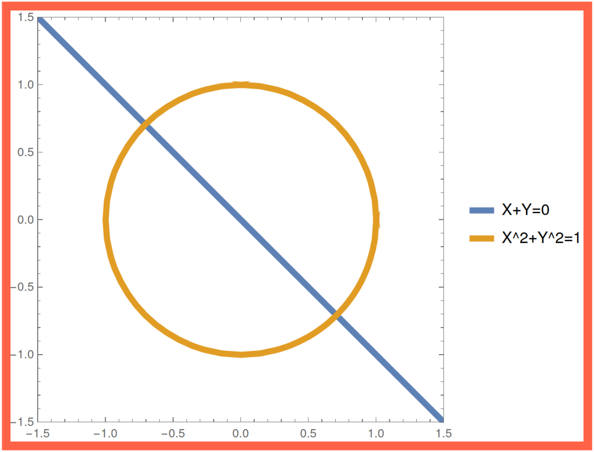
Or: The tropical Bézout's theorem

Line intersects line



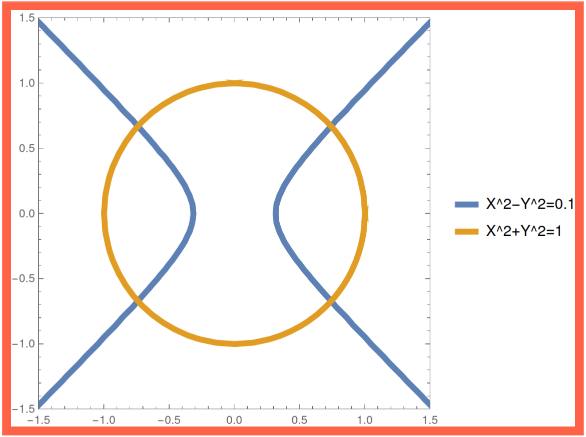
- Degree (d) one and one Two lines $p_1 \neq p_2$ intersect in one point $(1 \cdot 1 = 1)$
- Problem This is not quite true: they could be parallel
- Classical solution = invent projective space where this is always true

Line intersects circle



- d one and two A circle and a line intersect in two points $(1 \cdot 2 = 2)$
- Problem This is not quite true: the line could be "out of reach"
- Classical solution $=$ work over $\mathbb{C}$ and not $\mathbb{R}$

Hyperbola intersects circle



- d two and two A hyperbola and a circle intersect in 4 points $(2 \cdot 2 = 4)$
- Problem This is not quite true: they could be tangent to one another
- Solution $=$ say "intersect generically"

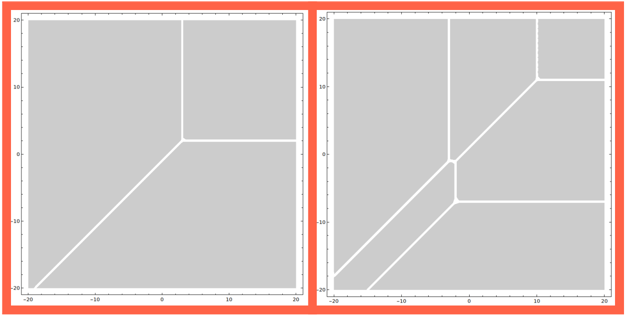
For completeness: A formal statement

Theorem

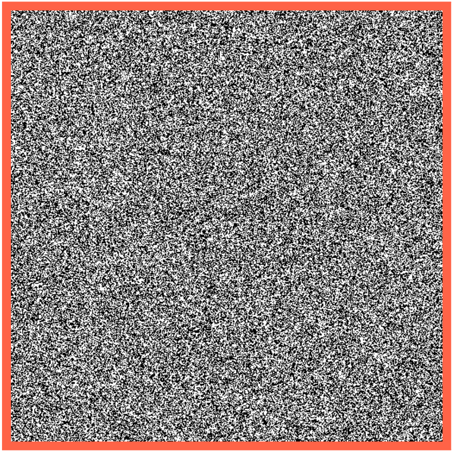
(i) Two complex projective curves intersect generically in ‘product of degrees’ many points

(ii) Two tropical curves intersect generically in ‘product of degrees’ many points

- Tropically there is no need to work over $\mathbb{C}$ or in projective space
- “Proof” Put the following pictures on top of one another $(1 \cdot 2 = 2)$:



Random coefficients



- Intersect generically = almost all of the time
- Example Choose coefficients randomly
- Above = a random image

Thank you for your attention!

I hope that was of some help.