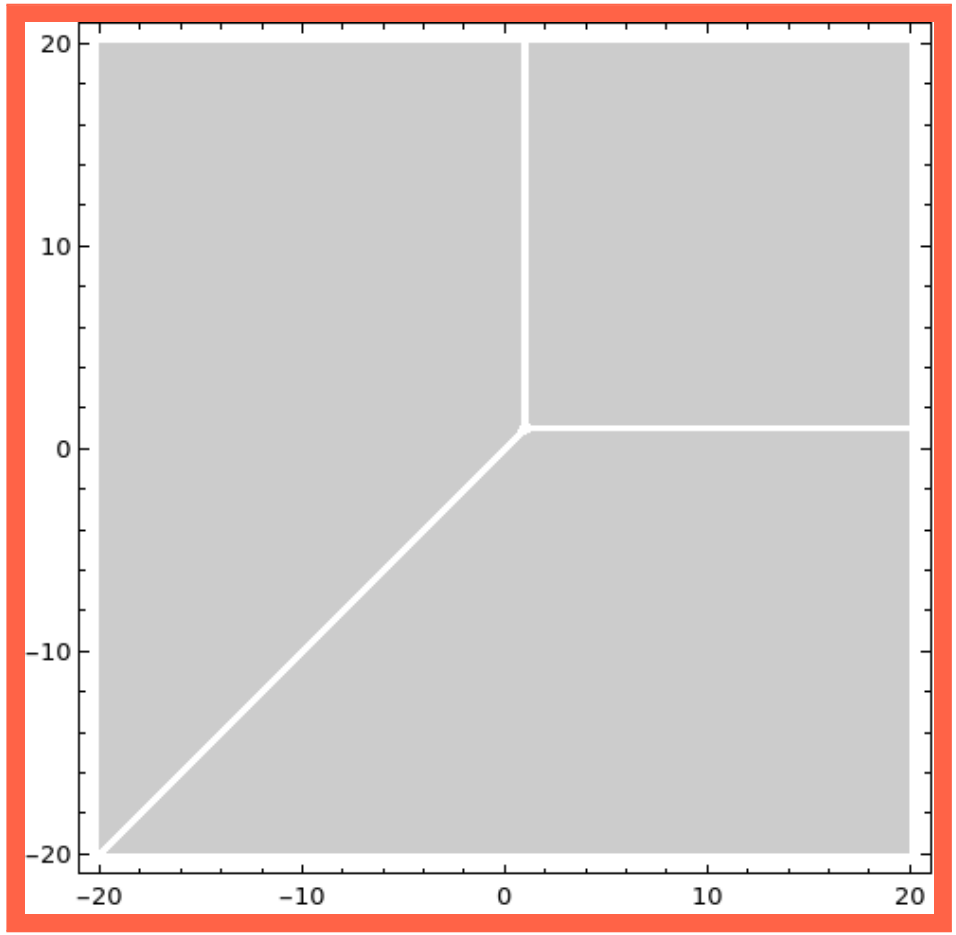
What is...tropical geometry - part 8?

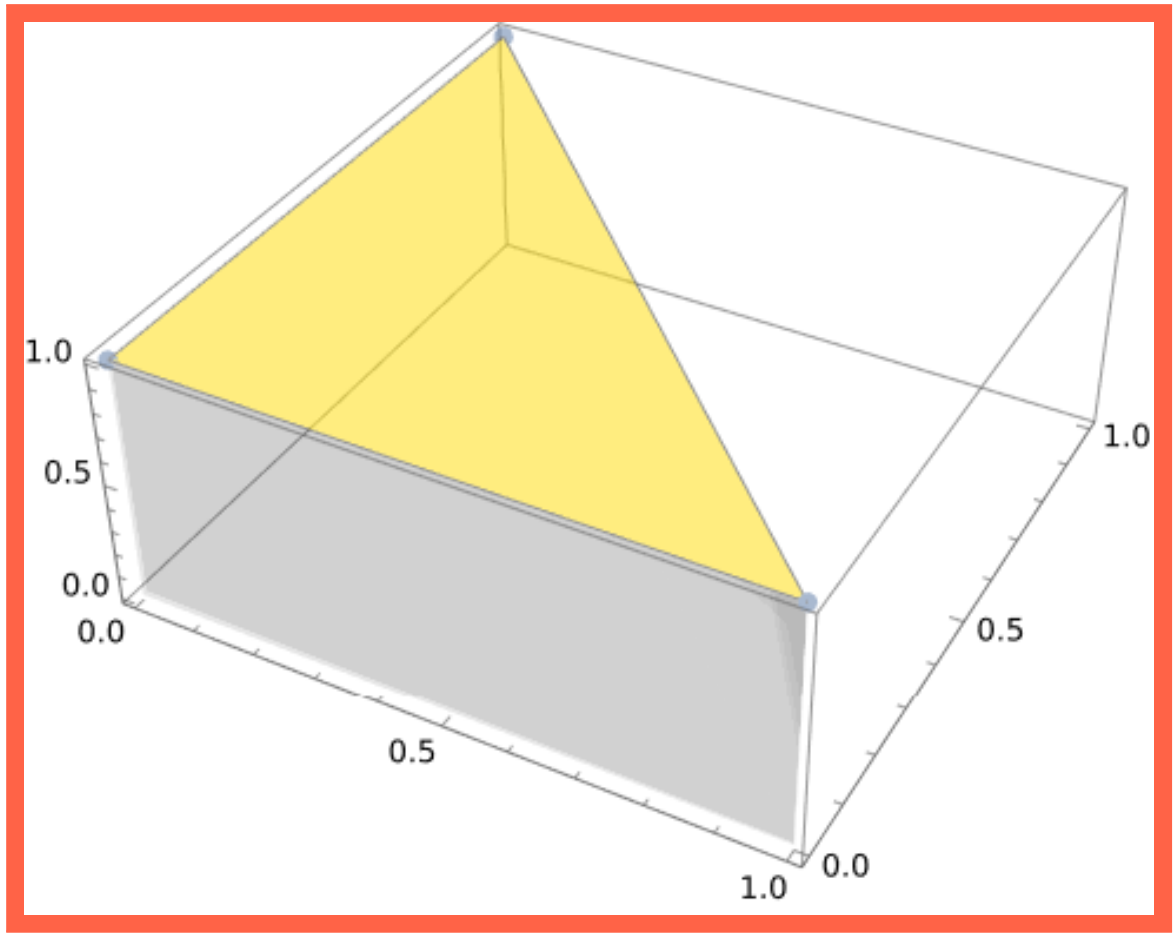
Or: Newton polygons

Tropical curves (reminder)



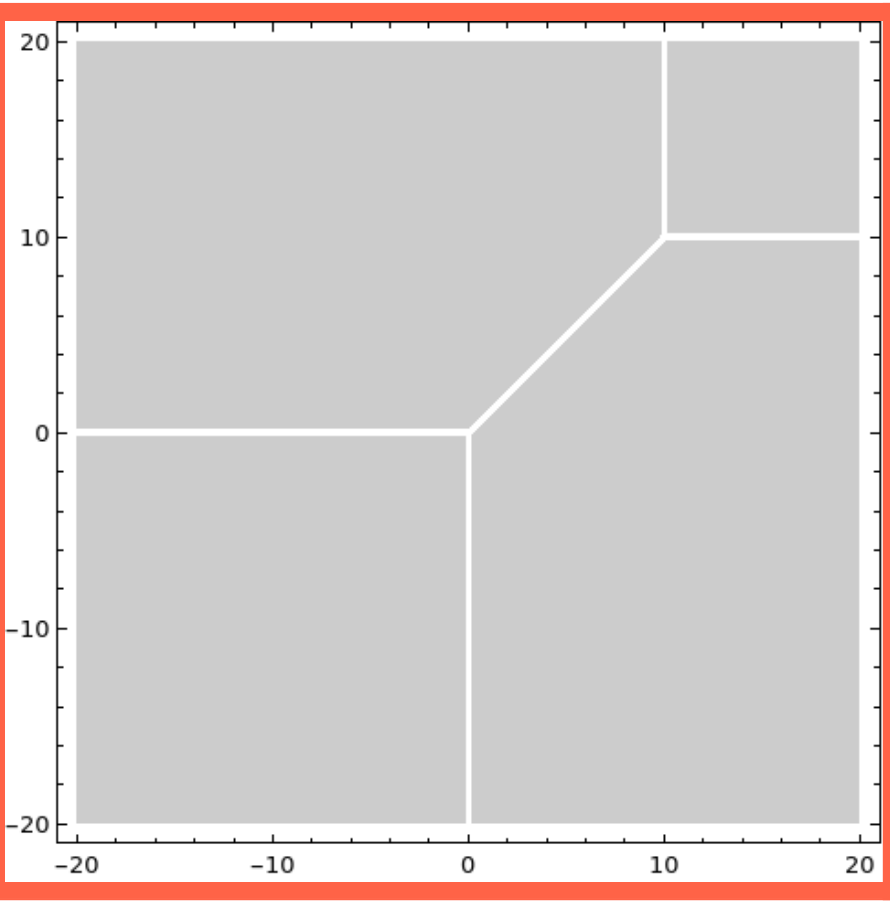
- Classical line = solutions of $ax + by + c = 0$
- Tropical line = tropical solutions (breaking points) of $\min\{a + x, b + y, c\}$
- Tropical curve = same but for higher degree polynomials

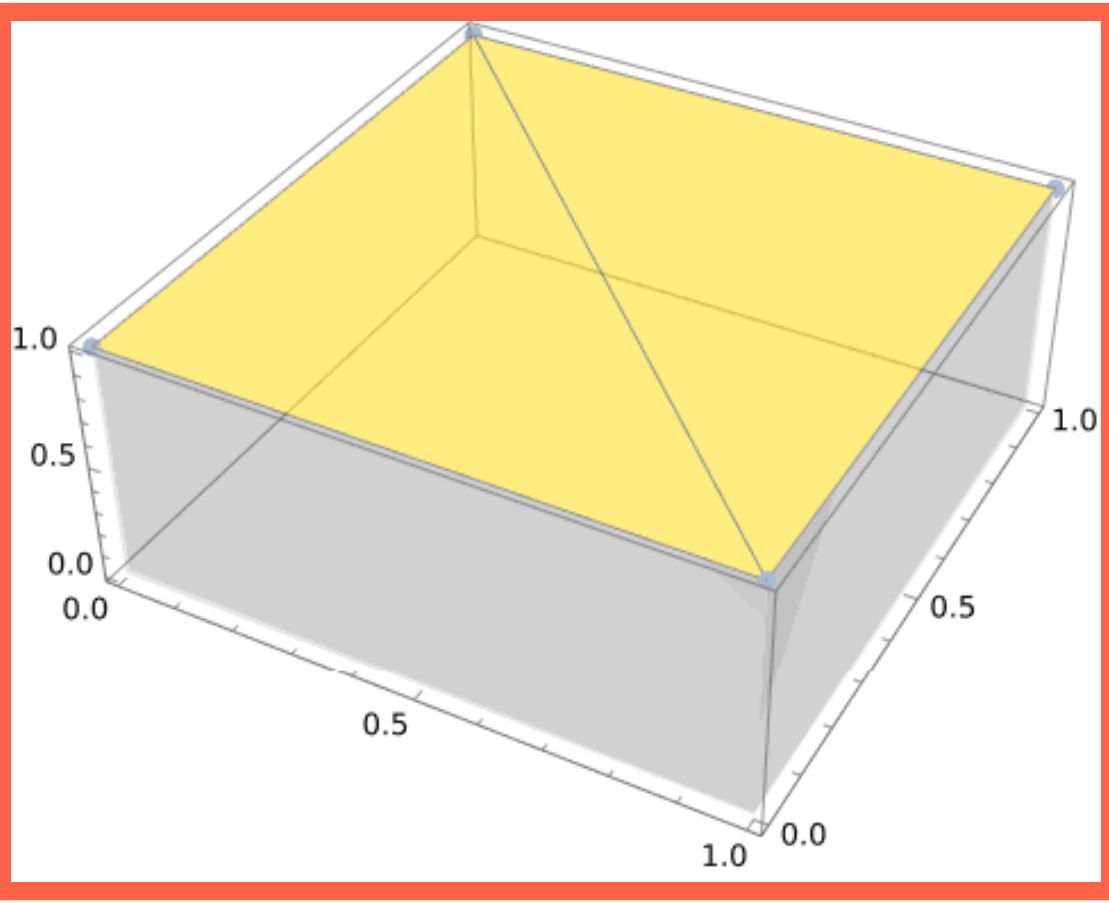
Look at the appearing powers



- Newton polygon (of a tropical curve) = the convex hull of the points (i, j) such that $x^i y^j$ appears in the classical polynomial for the tropical curve
- Above The Newton polygon of $x + y + 1 = 0$

The dual polygon





- Above A tropical curve and its Newton polygon
- They are dual!
- This is meant in the sense of dual planar graphs

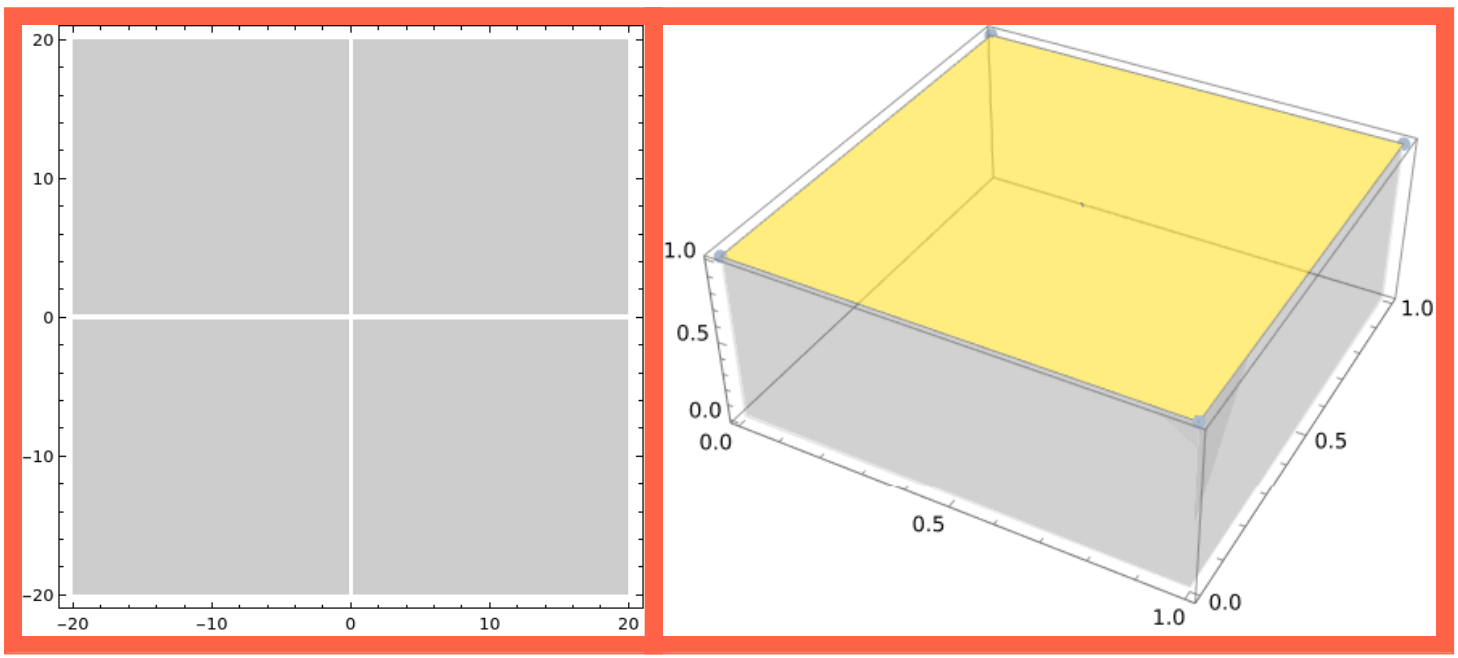
For completeness: A formal statement

Theorem

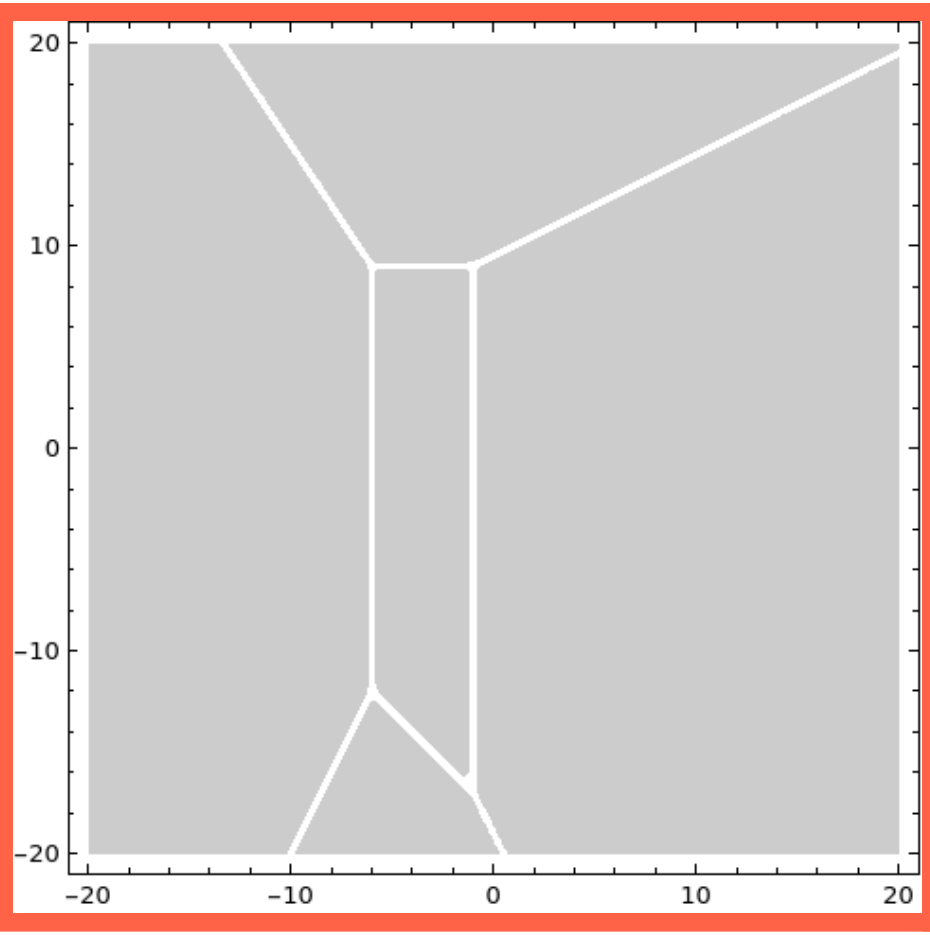
(i) Every tropical curve has an associated Newton polygon; every subdivision of a polygon gives a tropical curve "They are the same"

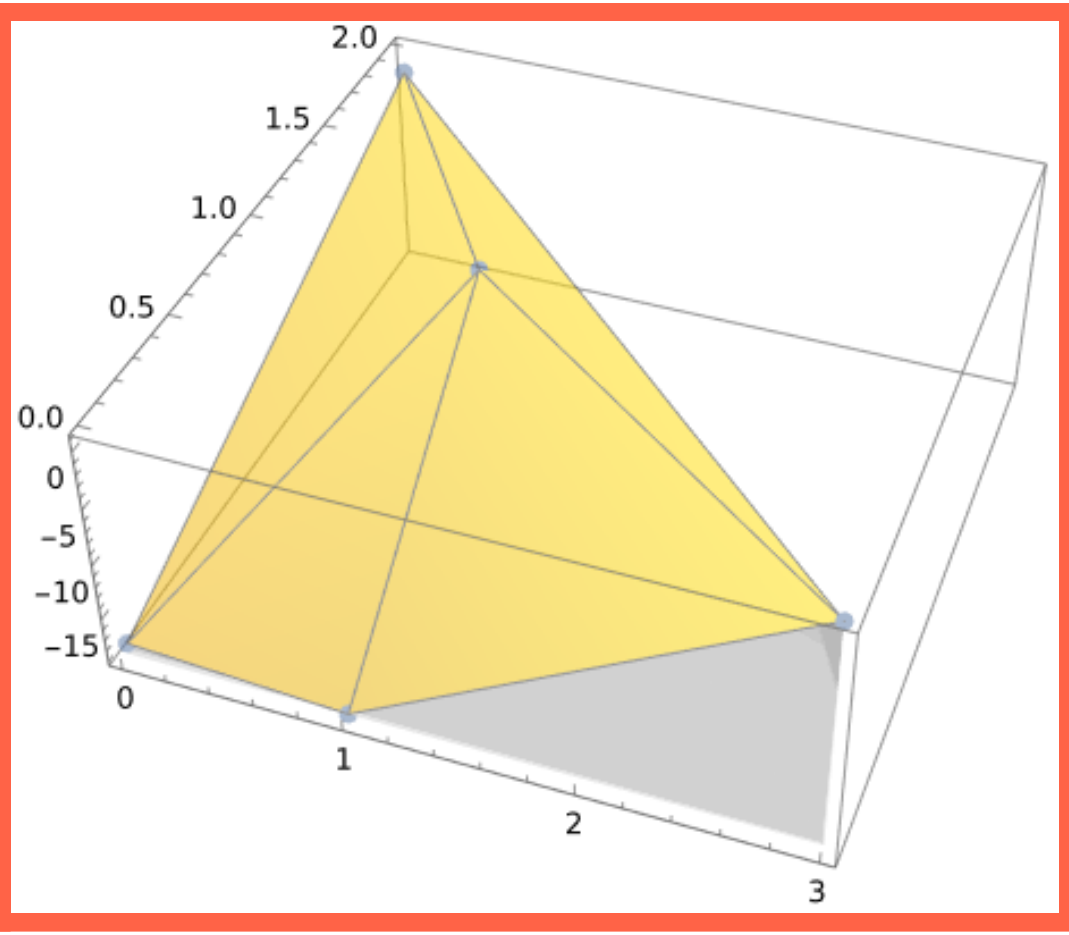
(ii) Polygons + sub division $\Leftrightarrow$ equivalence class of tropical curves

- Smooth = only trivalent vertices = subdivision is a triangulation
- A non-smooth example is:



Polygons and genus





- Above A tropical curve of genus one and its Newton polygon
- Genus = number of bounded faces
- Genus of the polygon = number of internal vertices

Thank you for your attention!

I hope that was of some help.