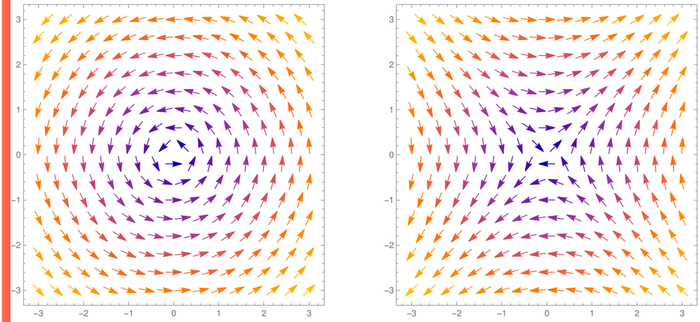
What is...tropical geometry - part 20?

Or: Tropical linear algebra 4 - Eigenvectors and polytropes

Reminder: Eigenvalues and eigenvectors

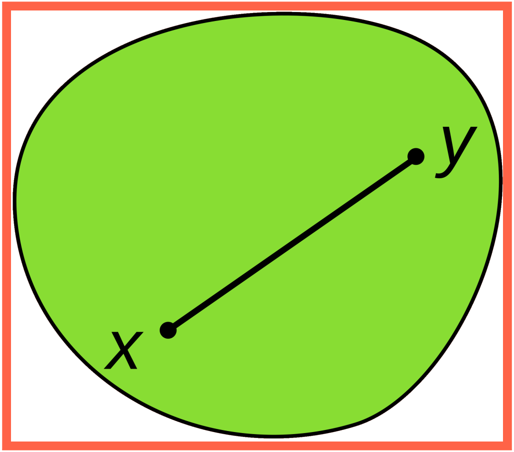
$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad\qquad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



no fixed vector — fixed vectors

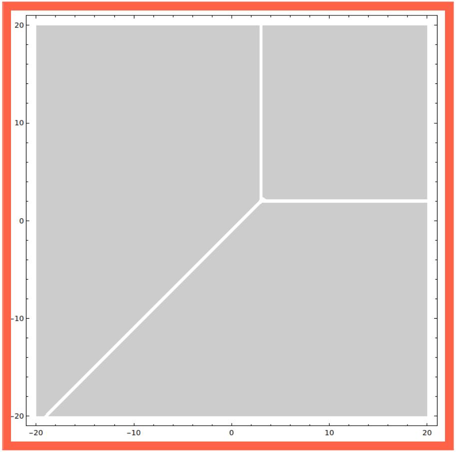
- **Above** An illustration of classical eigenvalues and eigenvectors
- **Recall** This means $Ax = \lambda \cdot x$, say $\lambda \in \mathbb{R}$
- The **same** definition applies for tropical matrices

Classical eigenspaces are convex



- Above A convex set
- Definition For any two points the line connecting them is in the set
- Question What happens tropically?

Tropical lines



- Recall A tropical line is not really a line, see above
- Definition Tropically convex = contains all tropical lines
- Polytrope is a space that is both, convex and tropically convex

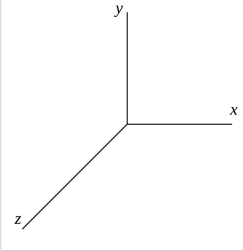
For completeness: A formal definition

The tropical eigenspace $eig(A)$ is tropically convex and satisfies

$$eig(A) = im(B_0^*)$$

for $B = -\lambda(A)A$, $B^* = B + \ldots + B^n$, B_0^* = columns of B^* with diagonal zero

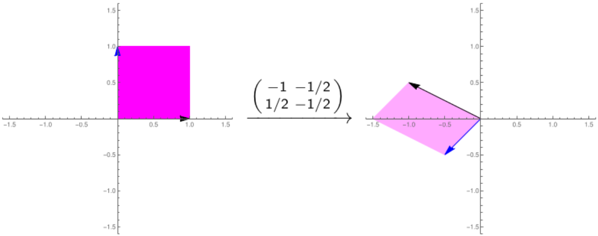
- Tropical eigenspace = solutions to $Ax = \lambda \cdot x$
- This implies that eigenspaces are images of matrices, so "planes"



Conventional visualization of the tropical projective plane, with projection of the real coordinate axes.

Determinants again

It is all about area



Wish. The determinant should be the area of matrix times unit square.

- **Classically** The eigenvalues of A are the roots of $\det(A + t\mathit{id})$
- **Tropically** The same works
- **Precisely** $\lambda(A) =$ smallest root of $\det(A + t\mathit{id})$

Thank you for your attention!

I hope that was of some help.