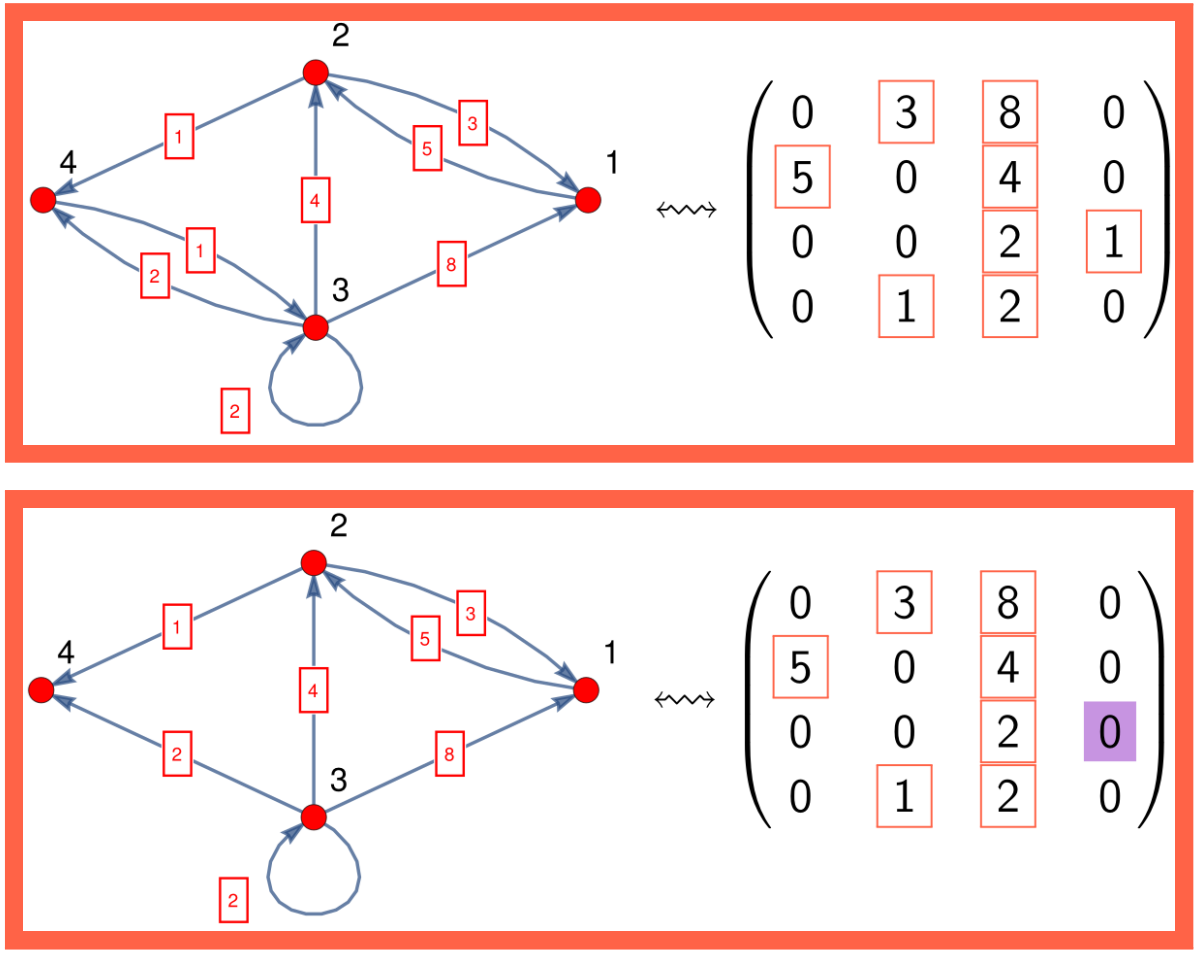
What is...tropical geometry - part 19?

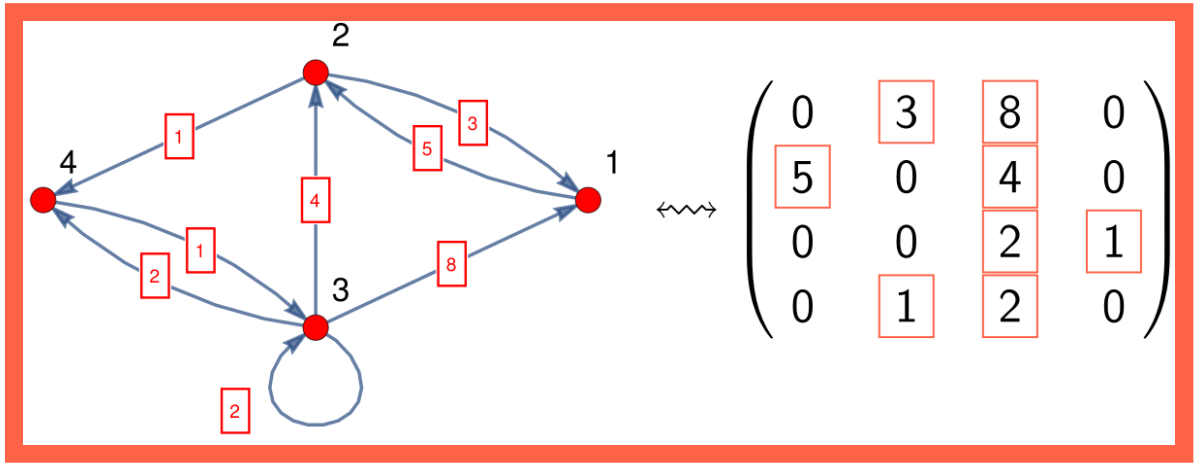
Or: Tropical linear algebra 3 - tropical matrix powers

Matrices and graphs



- Above "Why graphs and matrices are the same"
- Note No connection ⇜⇝ zero entry
- This is just the tip of the iceberg: matrices and graphs are deeply connected

Matrix powers and graphs

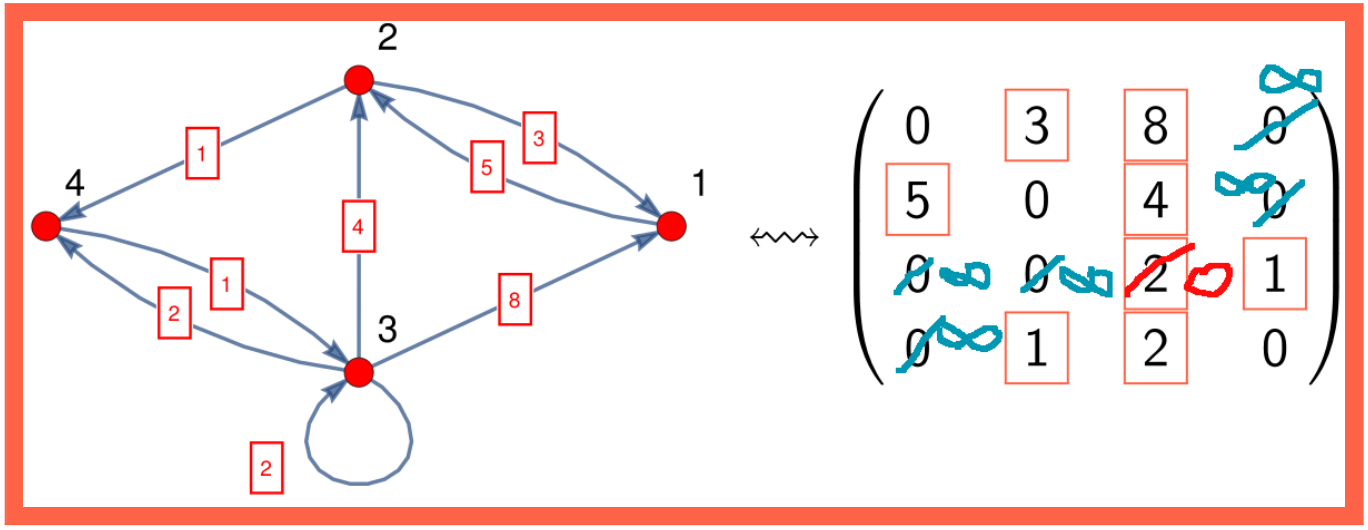


$$\leftrightsquigarrow \begin{pmatrix} 0 & 3 & 8 & 0 \\ 5 & 0 & 4 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 1 & 2 & 0 \end{pmatrix}$$

```
In[1]:= M := {{0, 3, 8, 0}, {5, 0, 4, 0}, {0, 0, 2, 1}, {0, 1, 2, 0}};

In[2]:= MatrixPower[M, 2] // MatrixForm

Out[2]//MatrixForm=
(15  0  28  8)
( 0 15  48  4)
( 0  1   6  2)
( 5  0   8  2)

In[3]:= MatrixPower[M, 3] // MatrixForm

Out[3]//MatrixForm=
( 0  53 192 28)
(75  4  164 48)
( 5  2  20   6)
( 0  17 60   8)
```

- Above The first few matrix powers of M
- Observation (and true) The entries of M^k correspond to # paths of length k
- Question What happens tropically?

Tropical matrix powers and graphs



$$\leftrightsquigarrow \begin{pmatrix} 0 & 3 & 8 & 0 \\ 5 & 0 & 4 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 1 & 2 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 3 & 8 & \infty \\ 5 & 0 & 4 & \infty \\ \infty & \infty & 0 & 1 \\ \infty & 1 & 2 & 0 \end{pmatrix}^2 = \begin{pmatrix} 0 & 3 & 7 & 9 \\ 5 & 0 & 4 & 5 \\ \infty & 2 & 0 & 1 \\ 6 & 1 & 2 & 0 \end{pmatrix}$$

- Slight change No connection $\leftrightsquigarrow$ ∞ entry, and diagonal $= 0$
- Observation (and true) The entries of M^k correspond to cost of the shortest paths of length $\leq k$
- This is like travel cost

For completeness: A formal definition

A tropical adjacency matrix M satisfies

(i,j)-entry of M^k = value of the cheapest path from i to j

- Tropical adjacency matrix = with ∞, and 0 on the diagonal
- This implies: $M^{k+1} = M^k$ for some k; like an idempotent

Application

- **Task** Find the price of the cheapest flight from A to B
- **Preparation** Write down the tropical adjacency matrix M of flight costs
- **Answer** The price is the (A,B)-entry of M^k for some k big enough

Thank you for your attention!

I hope that was of some help.