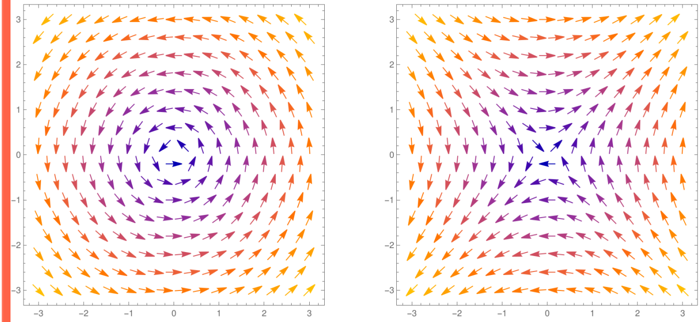
# What is...tropical geometry - part 17?

Or: Tropical liner algebra 1 - eigenvalues and graphs

# Eigenvalues and eigenvectors

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad\qquad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$




no fixed vector | fixed vectors

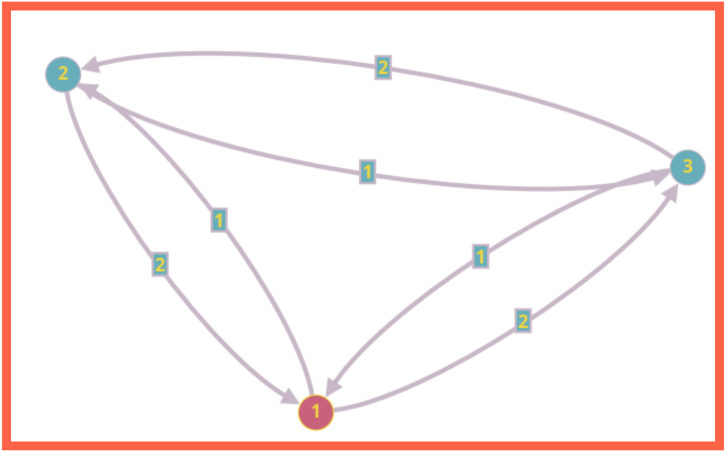
- **Above** An illustration of classical eigenvalues and eigenvectors
- **Recall** This means $Ax = \lambda \cdot x$, say $\lambda \in \mathbb{R}$
- The **same** definition applies for tropical matrices

$$\begin{pmatrix}0 & 1 & 0\\ 1 & 1 & 1\\ 0 & 1 & 0\end{pmatrix} = \begin{tikzcd} \mathbb{1} \arrow[r, shift left] & \mathrm{L}_s \arrow[l, shift left] \arrow[r, shift left] \arrow[loop below] & \mathrm{L}_{1'} \arrow[l, shift left] \end{tikzcd}$$

$$\begin{pmatrix}0 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 0\end{pmatrix} = \begin{tikzcd} \mathbb{1} \arrow[rr, bend left] & \mathrm{L}_s \arrow[loop below] & \mathrm{L}_{1'} \arrow[ll, bend right] \end{tikzcd}$$

- Recall Every matrix corresponds to an oriented graph, its adjacency graph
- There are two examples of this above
- Bigger entries correspond to labeled (=multiple) edges

## Oriented cycles and more




- Terminology 1 Strongly connected = connected in the oriented sense
- Terminology 2 Oriented cycles = cycles in the oriented sense
- Above The graph is strongly connected and has plenty of oriented cycles

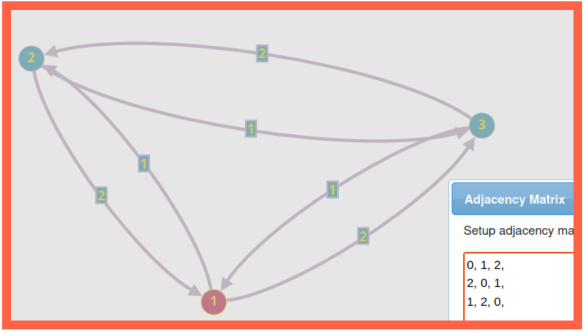
## For completeness: A formal definition

A tropical matrix with strongly connected adjacency graph has precisely one eigenvalue $\lambda(A)$ and

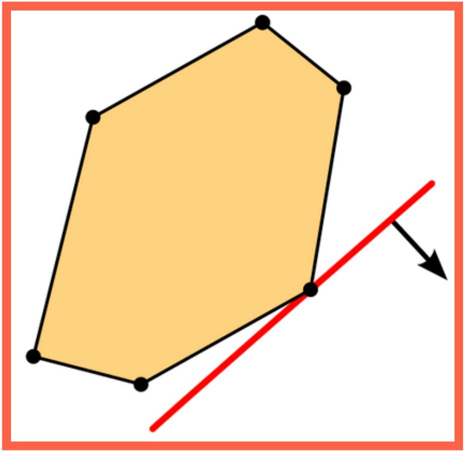
$$\lambda(A) = \text{minimal normalized length of any directed cycle}$$

(This means the sum of the labels of the edges divided by the length of the path)

- Example For the matrix below we have $\lambda(A) = 3/2$
- Why? There is an oriented cycle of length 2 and sum 3



## Linear programming again



- Linear program Maximize $\gamma$ subject to $a_{ij} + v_j \geq \gamma + v_i$ for all $1 \leq i, j \leq n$
- Notation $A = (a_{ij})_{i,j=1}^n$, $v_1, \ldots, v_n$ are decision variables
- Theorem $\lambda(A)$ coincides with the optimal value of this linear program

**Thank you for your attention!**

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I hope that was of some help.