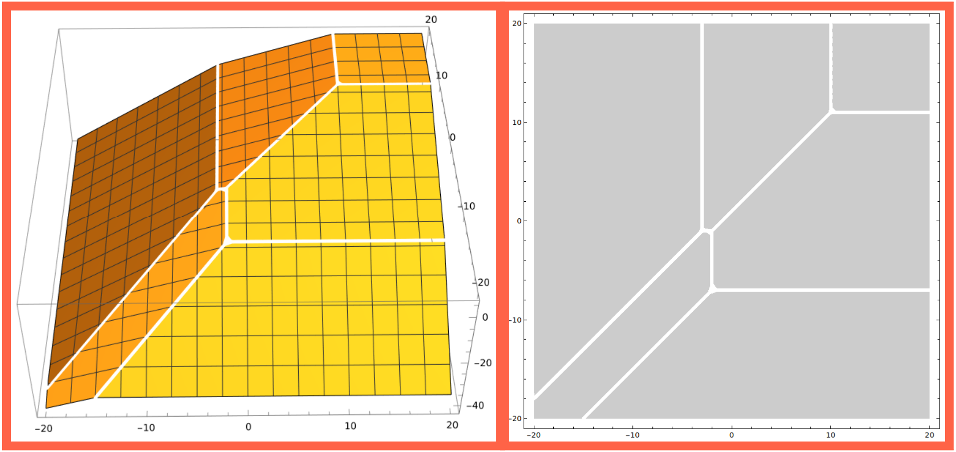
What is...tropical geometry - part 16?

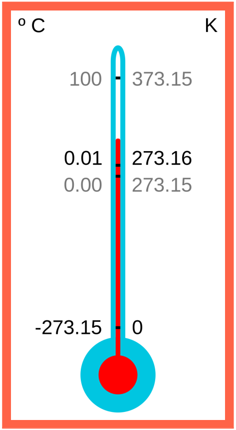
Or: Initial terms

Tropical varieties times 3



- We have seen three descriptions of a tropical variety
- Namely as "breaking points", as "min achieved twice" and as "log(variety)"
- This video Another method to get tropical varieties

Absolute zero

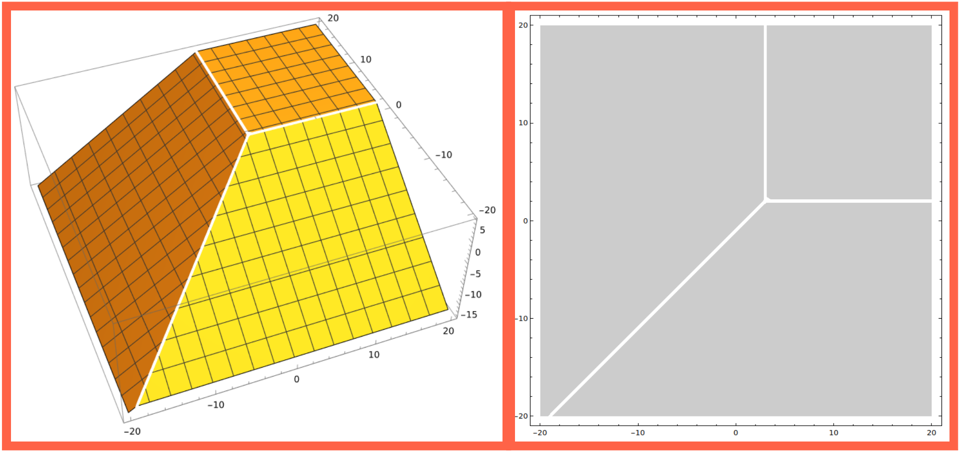


- Define the initial term $in_w(p)$ for $w \in \mathbb{Z}^n$ as

  $$p(t^{w_1}x_1, \ldots, t^{w_n}x_n), \text{ “scale by lowest power inverse” and set } t = 0$$

- Example $p = x^2 + y$, $w = (2, 3)$, then consider $p(t^2x, t^3y) = t^4x^2 + t^3y$, multiplying by t^{-3} gives $tx^2 + y$, and $t = 0$ gives $in_{(2,3)}(p) = y$

Absolute zero and tropical varieties



- Example The tropical line for $p = x + y + 1$
- $in_w(p)$ is a monomial unless w is a positive multiple of $(1, 0), (0, -1), (-1, -1), (0, 0)$
- These four points backbone of the above tropical line

For completeness: A formal definition

For a Laurent polynomial $p = \sum_{u \in \mathbb{Z}^n} c_u x^u$, the following two sets coincide:

- The tropical variety in $\mathbb{R}^n$ associated to p
- The closure in $\mathbb{R}^n$ of the set

$$\{w | in_w(p) \text{ is not a monomial}\}$$

- Warning I have not told you how to define a tropical variety in this generality, but we will get there
- There is a third part (omitted) about the amoeba

A classic in geometry

- The theorem is due to Kapranov ~1990 (unpublished work)
- In essence If you take a polynomial, find its zeros (a hypersurface), and then tropicalize both the polynomial and the points on the hypersurface, the tropicalized points will lie on the tropical variety of the tropicalized polynomial

Thank you for your attention!

I hope that was of some help.