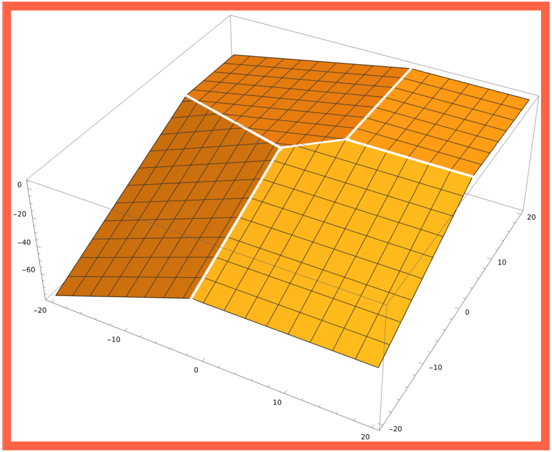
What is...tropical geometry - part 15?

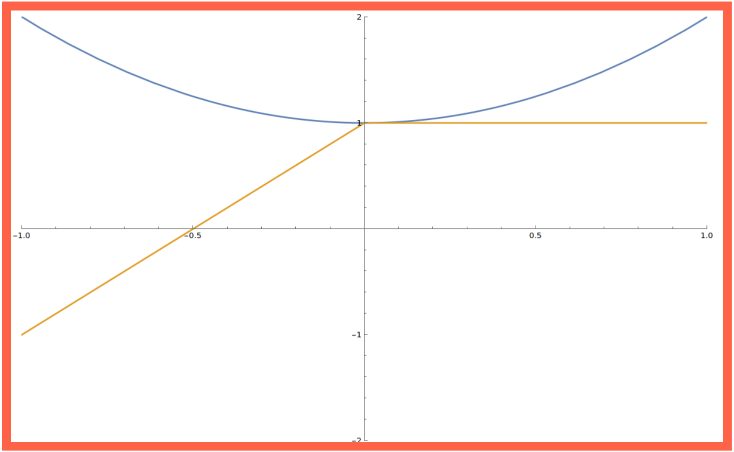
Or: Tropicalization

Tropical polynomial



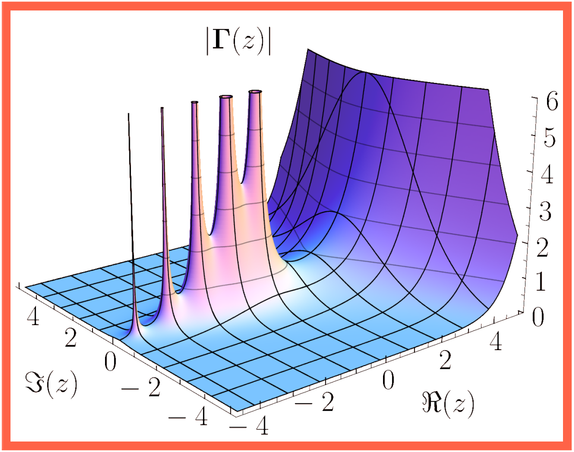
- **Above** The tropical polynomial $2x^2y^2 - y^2 - x + 4$, seen as a function $\mathbb{R}^2 \to \mathbb{R}$
- **Correct** This is $2 \otimes x \otimes x \otimes y \otimes y \oplus (-1) \otimes y \otimes y \oplus (-1) \otimes x \oplus 4$
- **Translated** This is $\min\{2 + 2x + 2y, -1 + 2y, -1 + x, 4\}$

Make it tropical



- Blue (top) curve above $x^2 + 1$
- Orange curve above $x^{\otimes 2} \oplus 1 = \min\{2x + 1, 1\}$
- Make it tropical $= + \mapsto \min, \circ \mapsto +$ (this maps 'poly $\mapsto$ tropical poly')

A better way to say this!?



- Tropicalization = take a log
- This is a bit like taking a valuation
- Example $\mathbb{C}\{\{t\}\}$ has a valuation given by $v(f) =$ the lowest exponent

For completeness: A formal definition

Given $p = \sum_{u \in \mathbb{N}^n} c_u x^u$ a polynomial, then

$$trop(p)\colon \mathbb{R}^n \to \mathbb{R}, w \mapsto \min(v(c_u) + w \cdot u)$$

is the tropicalization of p for a given valuation v

- Example For $p = (t + t^2)x_1 + 2t^2 x_2 + 3t^4 x_3 \in \mathbb{C}\{\{t\}\}[x_1, x_2, x_3]$, we have $trop(p) = \min(1 + w_1, 2 + w_2, 4 + w_3)$
- Logs everywhere!

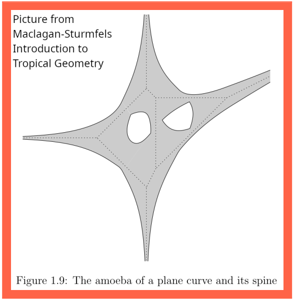
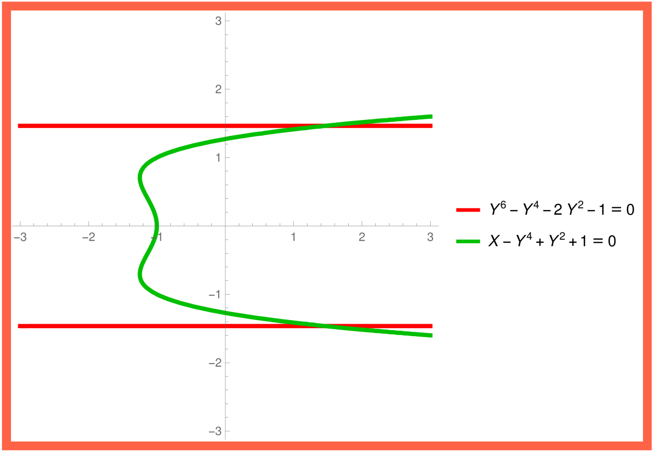


Figure 1.9: The amoeba of a plane curve and its spine

On our way to Gröbner bases



- Above The intersection of two varieties
- Simultaneously solving polynomial equations can be done with Gröbner bases
- TG plays a crucial role in this process

Thank you for your attention!

I hope that was of some help.