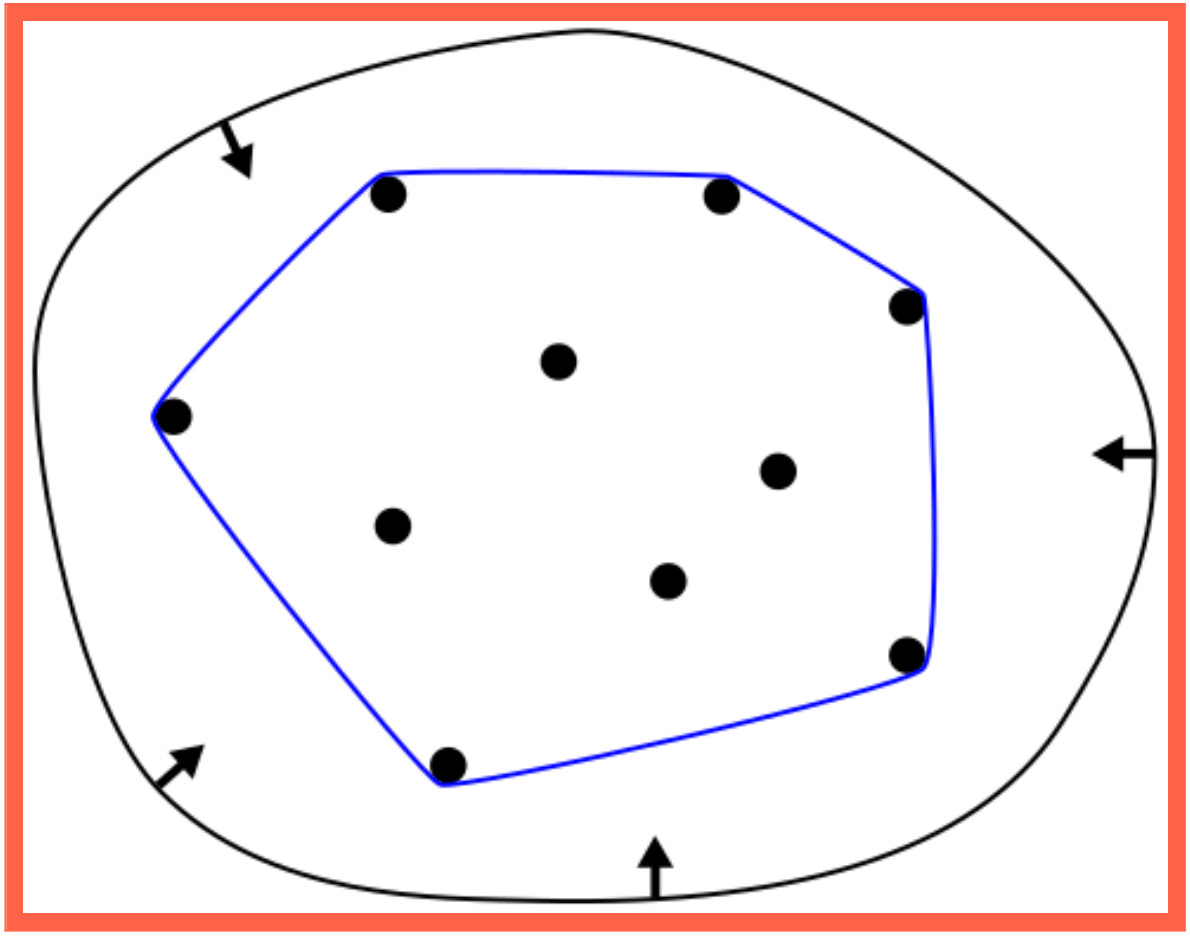
What is...tropical geometry - part 14?

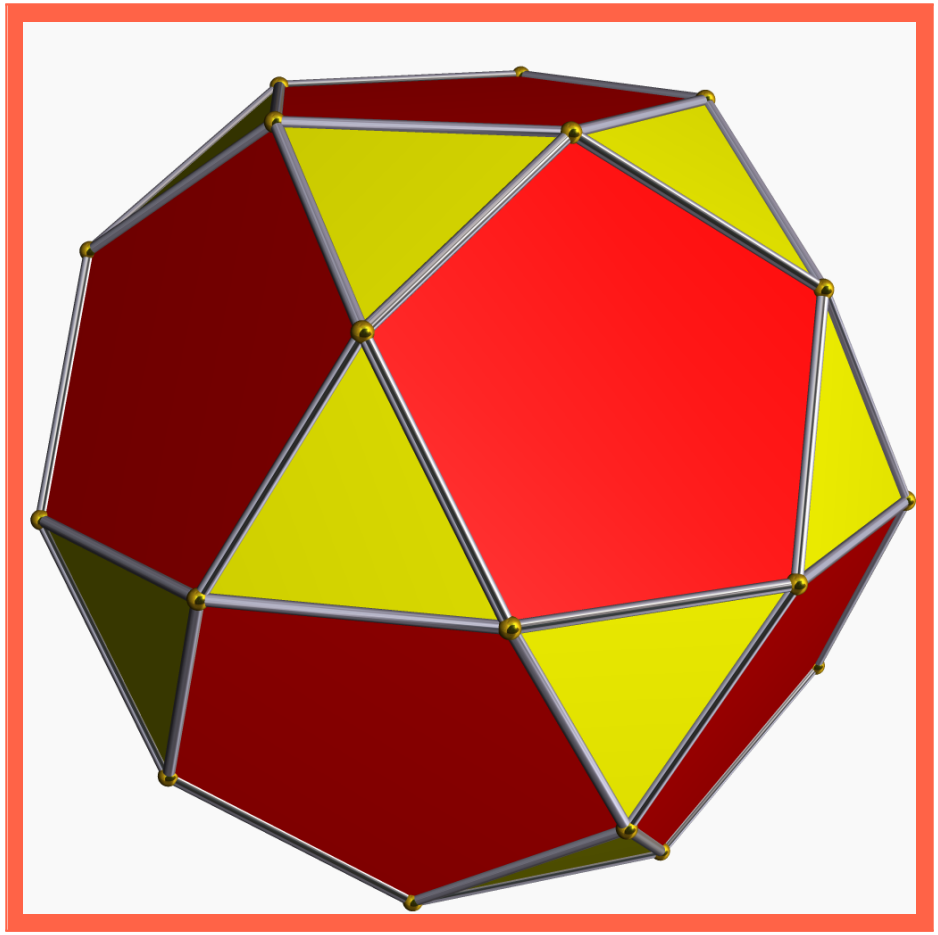
Or: Polyhedral geometry

Convex hull



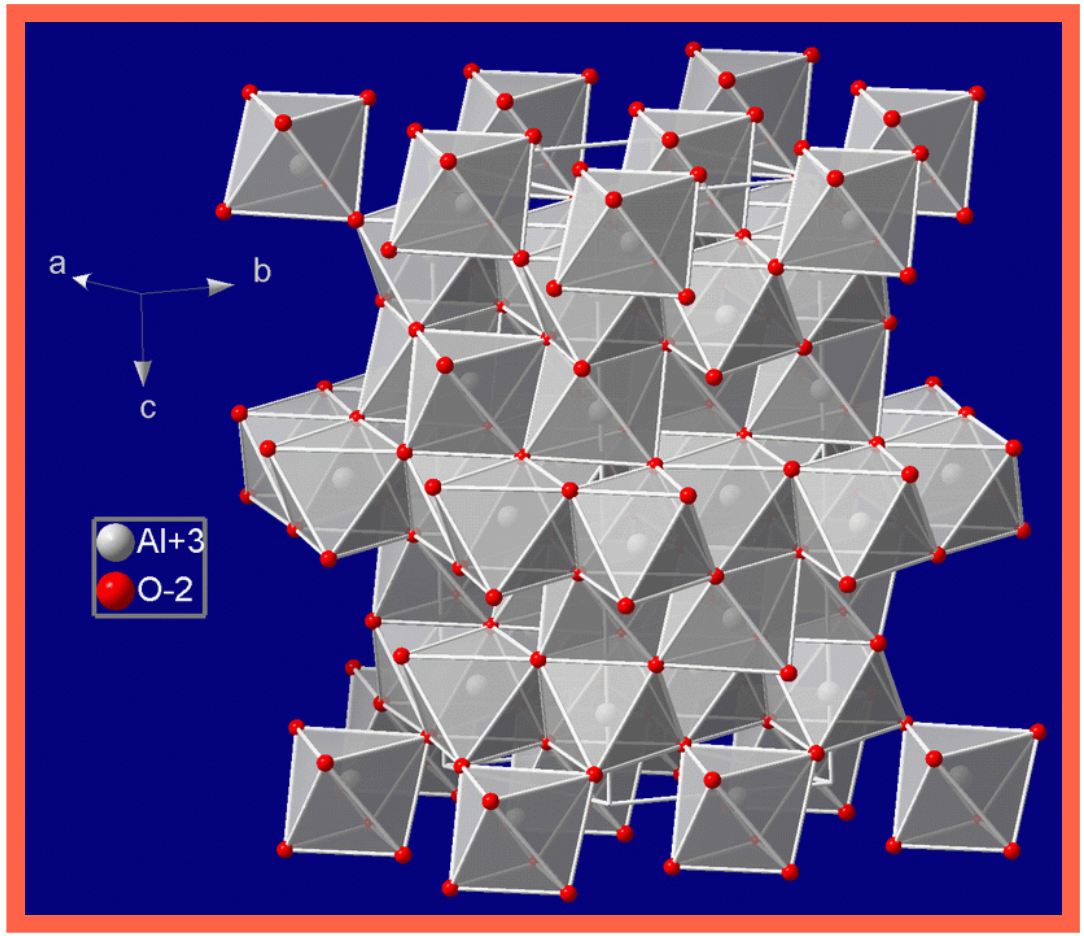
- Above The convex hull of a point cloud (a polyhedron)
- This is a standard notion in polyhedral geometry
- TG is piecewise linear so crucially related to polyhedra

A collection of linear inequalities



- Above A polytope (= bounded polyhedron)
- Definition A polyhedron $P \subset \mathbb{R}^n$ = intersection of finitely many closed half-spaces
- In formulas $P = \{x \in \mathbb{R}^n | Ax \leq b\}$

Many polyhedra together



- Above A polyhedral complex (almost)
- There are two conditions: if a polyhedron P is in the collection, then so is any face of P, and if P and Q lie in the collection then $P \cap Q$ is a face of both P and Q

For completeness: A formal definition

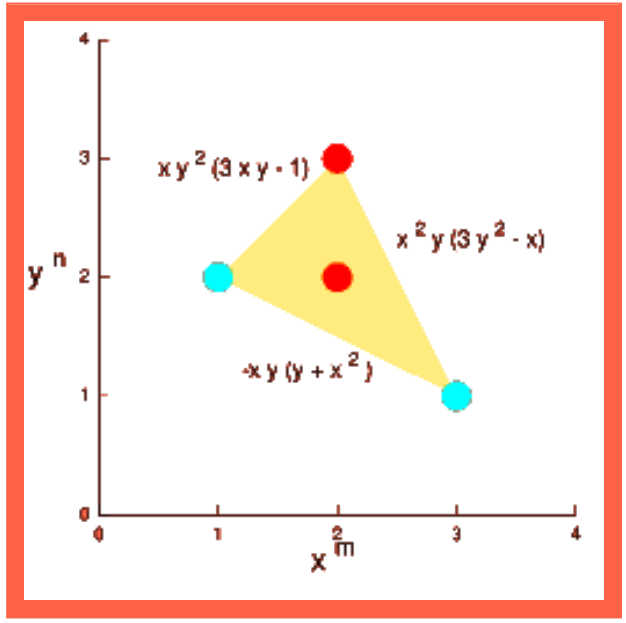
Given $p = \sum_{u \in \mathbb{Z}^n} c_u x^u$ a (Laurent) polynomial, then

$$N(p) = \text{convex hull of } \{u | c_u \neq 0\}$$

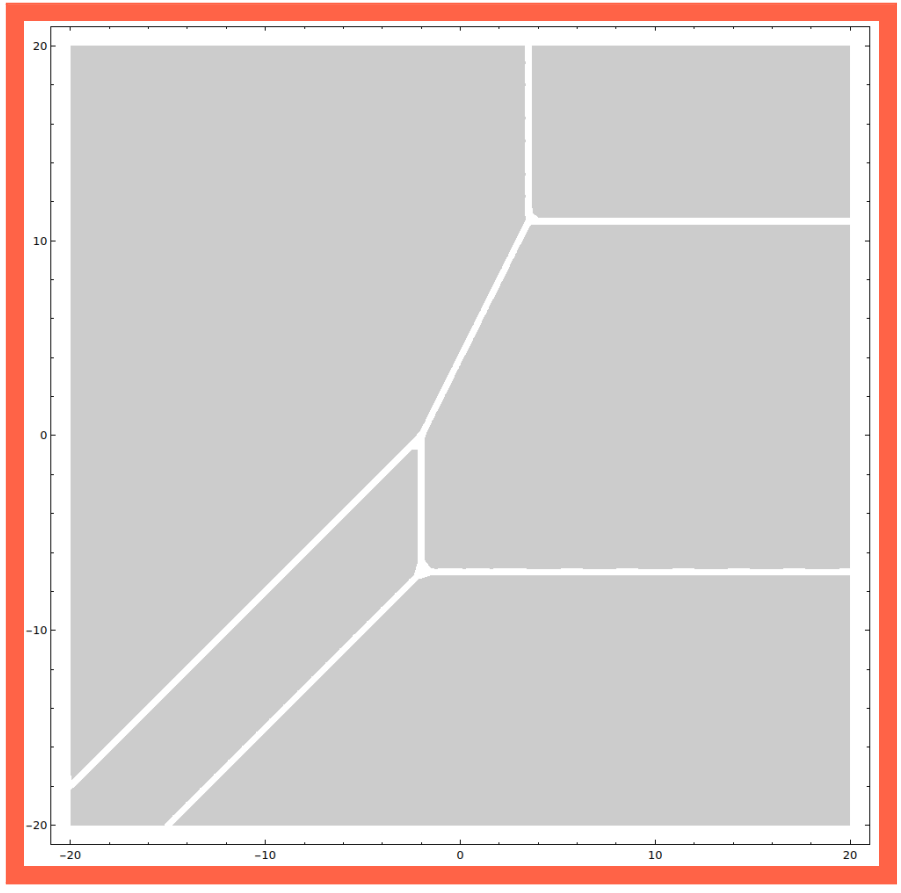
is the Newton polytope of p and we have

$$N(pq) = N(p) + N(q)$$

- Above we are using the Minkowski sum $A + B = \{a + b | a \in A, b \in B\}$
- Example The Newton polytope of $3x^2y^3 - xy^2 + 2x^2y^2 - x^3y$ is



Variety = polyhedron?



- Above A quadratic tropical curve
- This is an example of a polyhedral complex (made of line segments and rays)
- Together with the Newton polytope, this is another reason why TG and polyhedral geometry are closely related

Thank you for your attention!

I hope that was of some help.