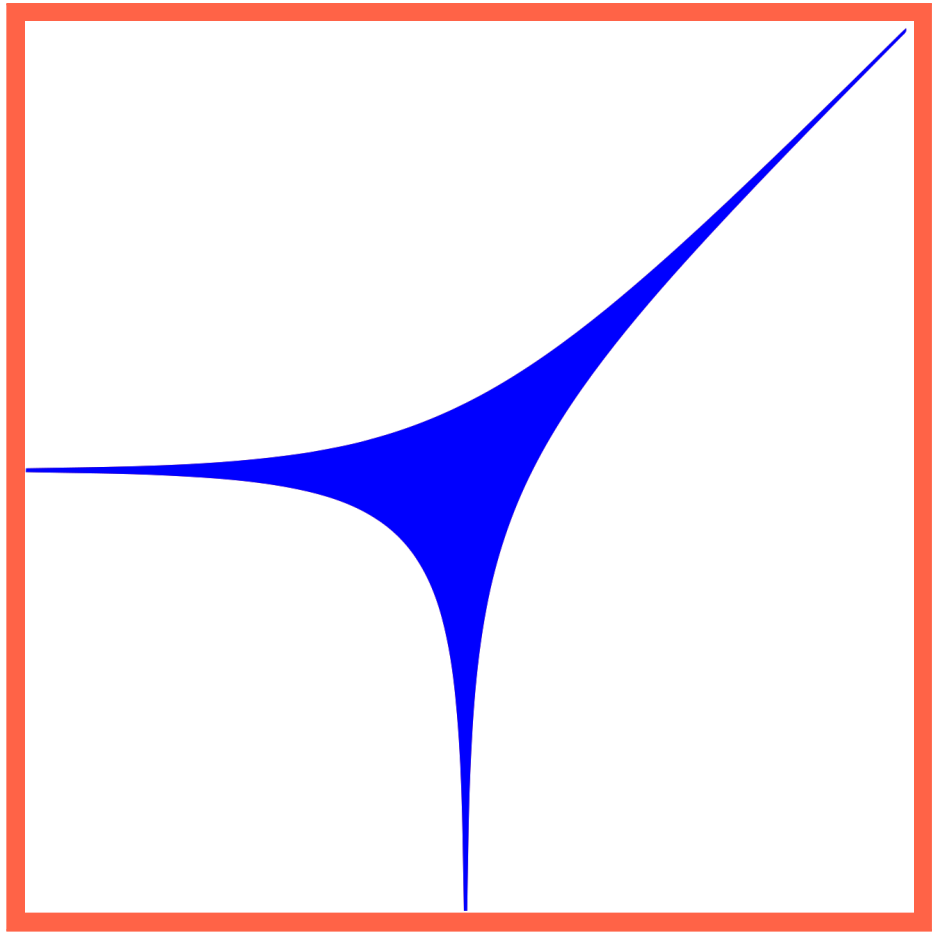
What is...tropical geometry - part 10?

Or: Amoebas

Tropical = logarithmic?

- Recall Tropically, multiplication turns into addition
- Recall Under a logarithm, multiplication turns into addition
- Above Amoeba proteus (we see in second why its relevant)

Amoebas in math



- Amoeba of a polynomial $f : \mathbb{C}^n \to \mathbb{C} =$ (log of the set of zeros of f)
- In other words Amoeba = (log of a variety)
- Above The amoeba of the line $f(x, y) = y - 2x - 1$

Skeletons

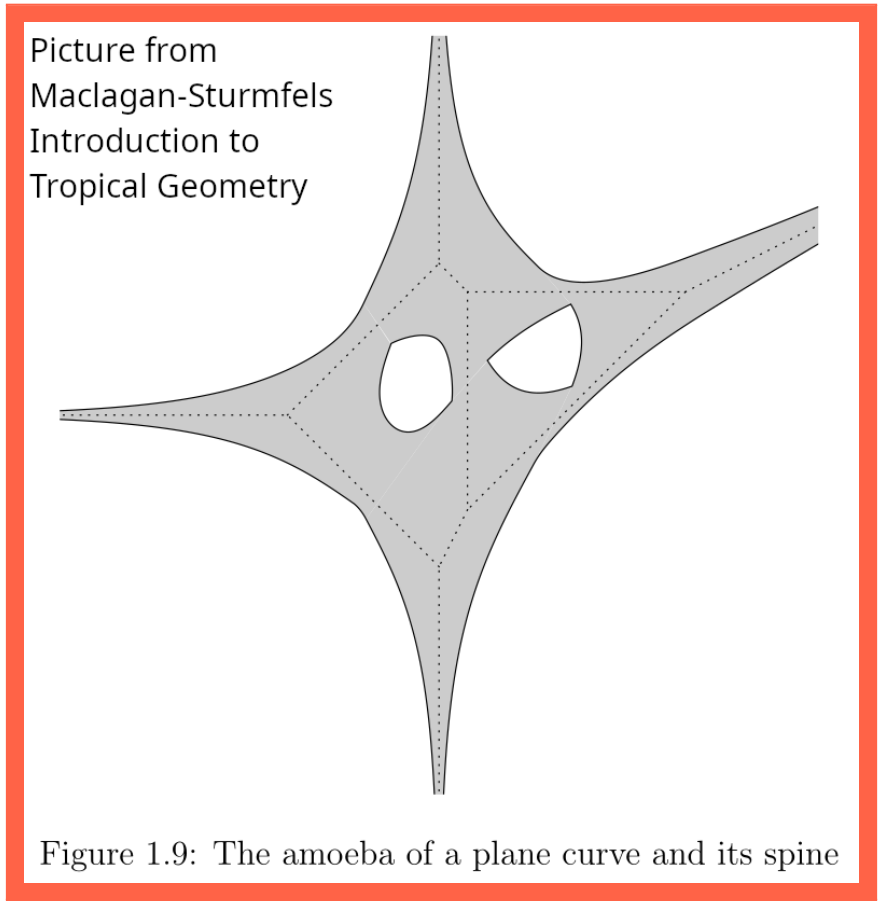


Figure 1.9: The amoeba of a plane curve and its spine

- The skeleton/spine of an amoeba is a tropical variety
- Hence "Tropical variety = log of a variety"
- Above An amoeba and its tropical variety

For completeness: A formal statement

The amoeba A_f of a complex polynomial f is:

$$A_f = \left\{ \left(-\log|z_1|, -\log|z_2|, ..., -\log|z_n| \right) : z = (z_1, ..., z_n) \in \left(\mathbb{C} \setminus \{0\} \right)^n, f(z) = 0 \right\}$$

(i) The area of $A_f, f \neq 0$ in two complex variables is finite

(ii) A 2d amoeba has some infinitely long and exponentially narrow tentacles

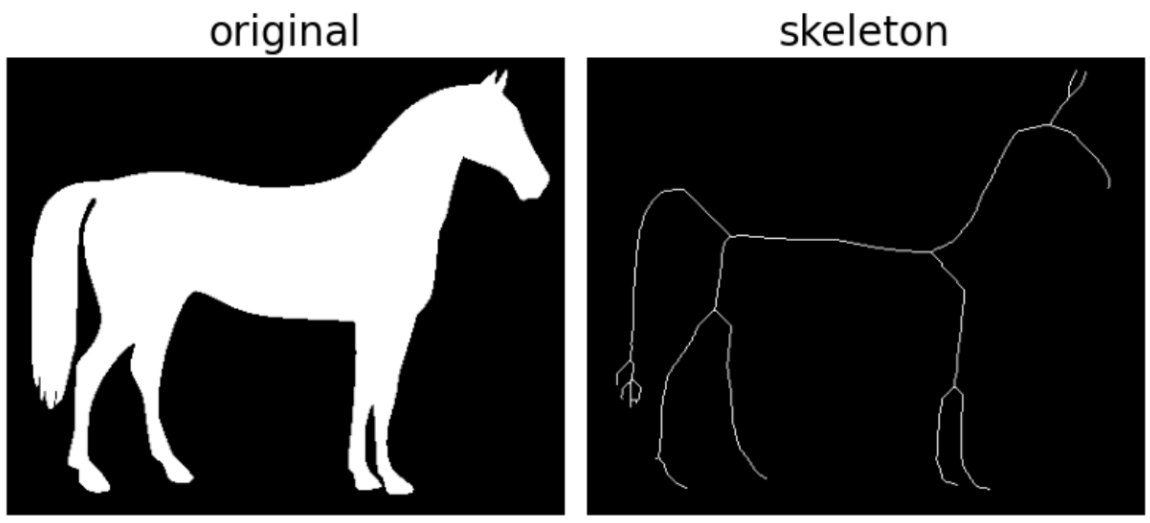
- Convention Since $\oplus = \min$ and not max, we meed a sign in the logarithm
- Example A genus one amoeba:

Skeletons of images

```
ax[0].imshow(image, cmap=plt.cm.gray)
ax[0].axis('off')
ax[0].set_title('original', fontsize=20)

ax[1].imshow(skeleton, cmap=plt.cm.gray)
ax[1].axis('off')
ax[1].set_title('skeleton', fontsize=20)

fig.tight_layout()
plt.show()
```



- Skeletonize = a standard procedure in image processing
- Tropical variety = skeletonize(log(variety))
- There is also an honest math formulation, but its omitted ☺

Thank you for your attention!

I hope that was of some help.