

What is...the Tsetlin library?

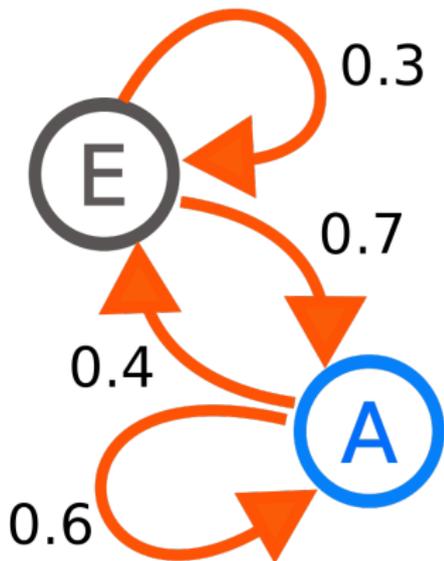
Or: Sorting books made easy

A lazy way of sorting book (I am a big fan!)



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- ▶ You need to keep a library organized, but have no time to do it
 - ▶ **Self-organizing system** A returned book gets placed to the right of all other books
 - ▶ Relative usage frequency \Rightarrow relative position
 - ▶ **Goal** Model and analyze this mathematically

Discrete Markov chains

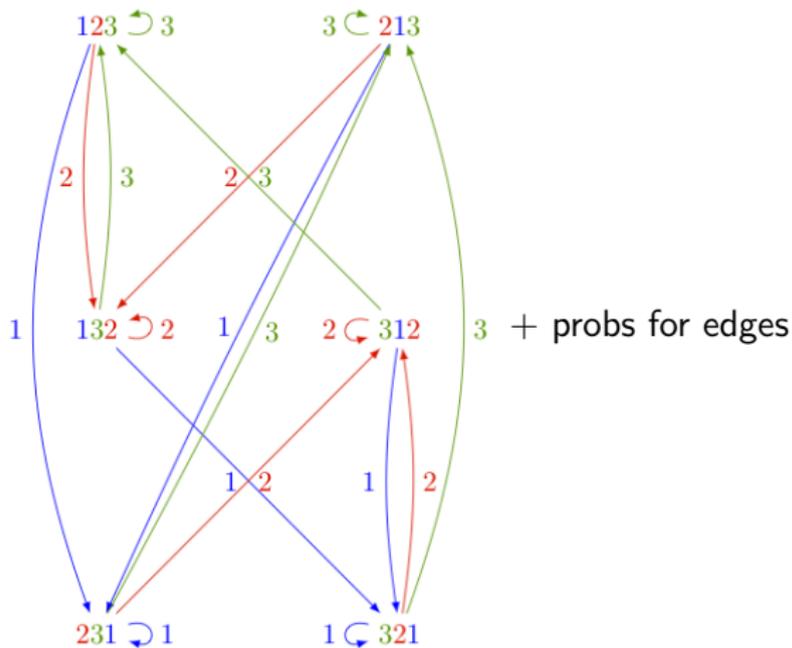


\Leftrightarrow

	A	E
A	0.6	0.7
E	0.4	0.3

- ▶ Discrete Markov chain “=” finite graph with weighted edges
- ▶ The outgoing edge sum is 1
- ▶ These can be analyzed using (column) stochastic matrices

Random walks



- ▶ This can be modeled using a random walk on Cayley graphs
- ▶ This time we have a Cayley graph of a non-group monoid
- ▶ **Strategy** Apply the same techniques as for group!

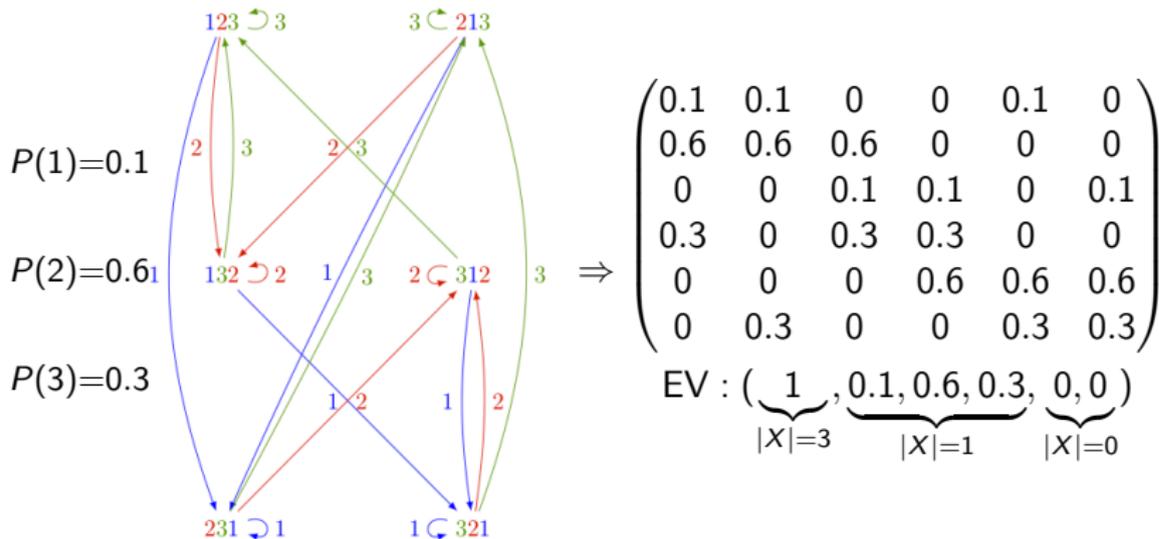
For completeness: A formal statement

The eigenvalues of the Tsetlin library random walk are

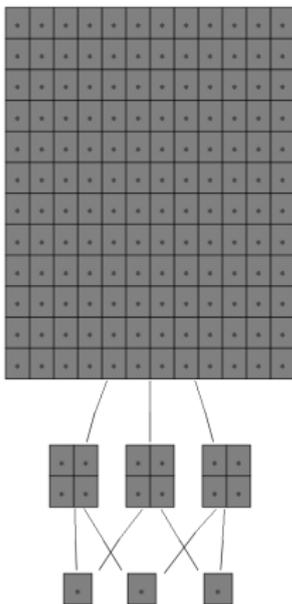
$$\lambda_X = \sum_{x \in X} P(x)$$

where $X \subset \text{books}$ and $P(x) = \text{prob of picking } x$

Every such EV appears $(\# \text{ fixed point free perm in } S_{|\text{books}| - |X|})$ times



Rep theory of bands



- ▶ A band is a semigroup in which every element is idempotent
- ▶ The Tsetlin library can be phrased using rep theory of a right regular band
- ▶ The Tsetlin library is a special case of random walks on monoids/semigroups

Thank you for your attention!

I hope that was of some help.