What are...the simples of the Brauer monoid?

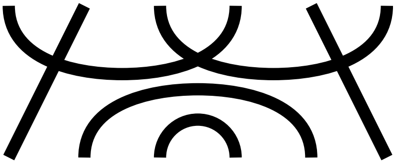
Or: Through strands

The H-reduction in action

$$\left\{\begin{matrix}\text{simples with}\\ \text{apex } \mathcal{J}(e)\end{matrix}\right\} \xleftrightarrow{\text{one-to-one}} \left\{\begin{matrix}\text{simples of (any)}\\ \mathcal{H}(e) \subset \mathcal{J}(e)\end{matrix}\right\}$$

Reps of monoids are controlled by $\mathcal{H}(e)$-cells

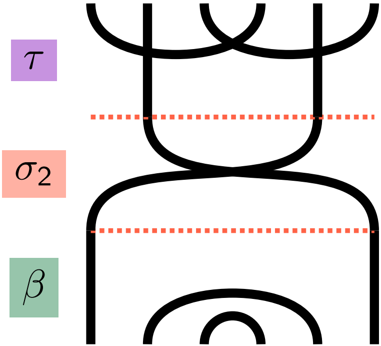
- We want to use this to classify simple reps for the Brauer monoid (see previous video)



$\in Br_6$

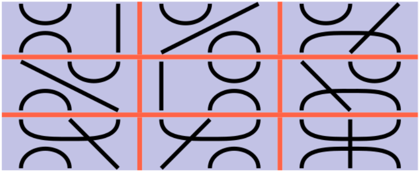
- To do list Find the J-cells, find idempotents and compute $\mathcal{H}(e)$-cells

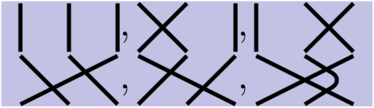
Bottom-middle-top



- Decompose Brauer diagrams into a bottom, a middle and a top part
- Bottom β Only caps and a minimal number of crossings
- Middle σ_λ Only crossings in some symmetric group S_λ on $\{1, \ldots, \lambda\}$
- Top τ Only cups and a minimal number of crossings

The cells



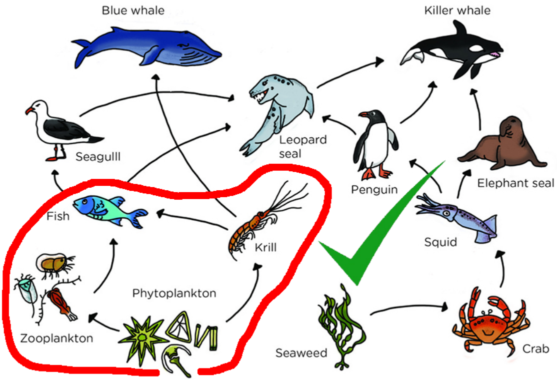


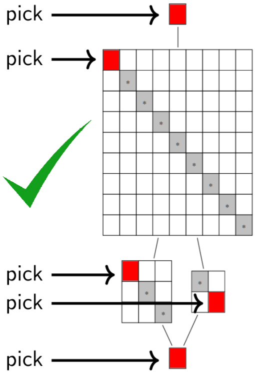
- Multiplication can only increase the number of through strands $\lambda \Rightarrow J$-cells are indexed by through strands λ
- Multiplication form the left = top does not change the bottom $\Rightarrow$ left cells have fixed bottom
- Multiplication form the right = bottom does not change the top $\Rightarrow$ right cells have fixed top
- Every $\updownarrow$-symmetric diagram is idempotent
- The middle is $S_\lambda \Rightarrow \mathcal{H}(e)$-cells are S_λ

For completeness: A formal statement

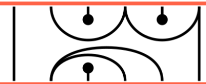
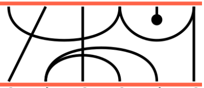
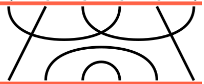
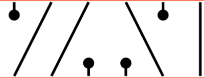
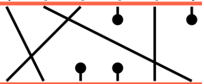
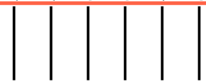
For reps of Br_n over $\mathbb{C}$ we get:

- The set of apexes is $\{n, n-2, n-4, \dots\}$ Through strands
- There are precisely (# partitions of λ) simples of apex λ $\mathcal{H}(e) \cong S_\lambda$





H-reduction and diagram monoids

Symbol	Diagrams	Symbol	Diagrams
pPa_n		Pa_n	
Mo_n		RoBr_n	
TL_n		Br_n	
pRo_n		Ro_n	
pS_n		S_n	

- The same works for many more diagram monoids
- Doing this is a fun exercise!

Thank you for your attention!

I hope that was of some help.