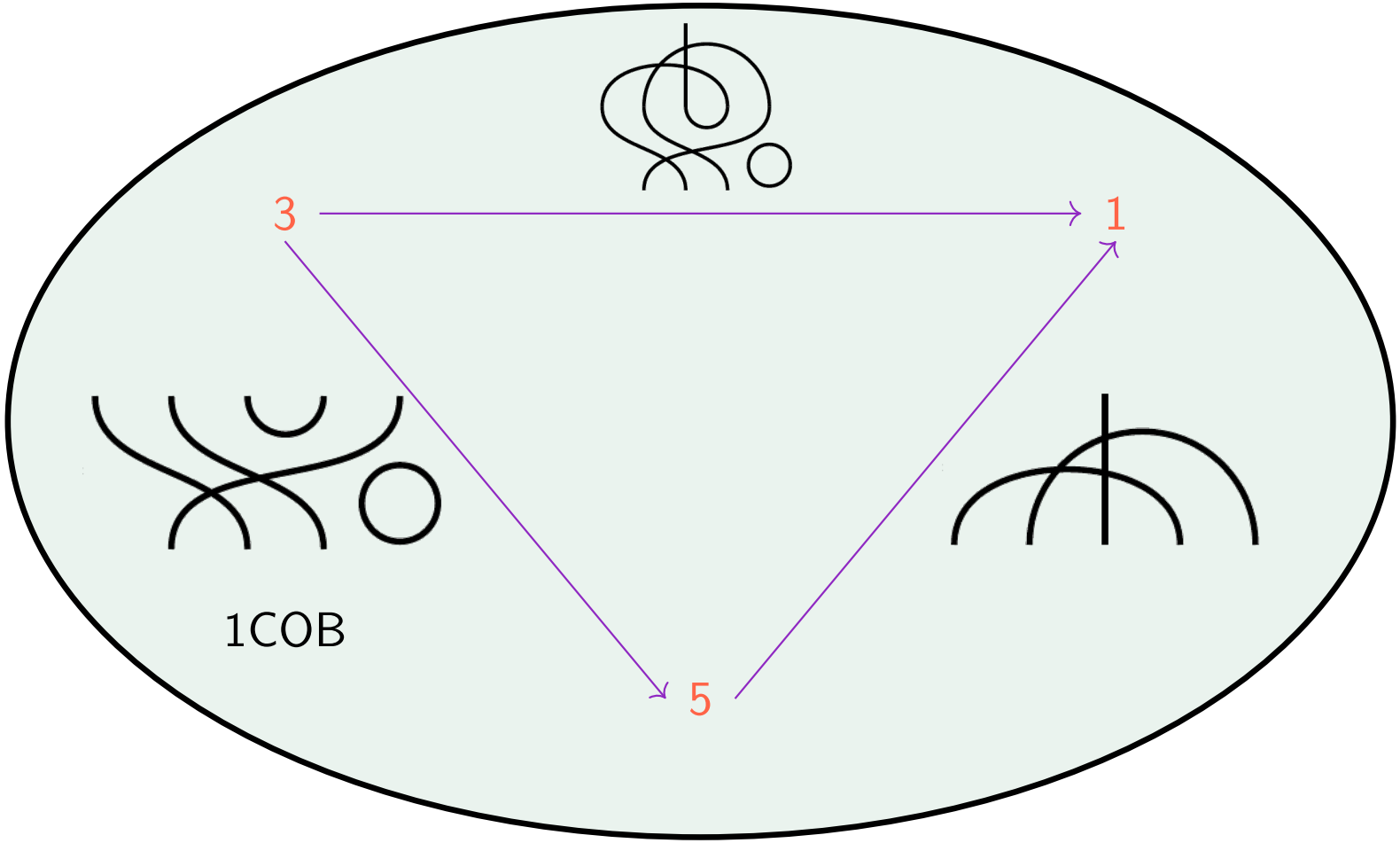
What is...the Brauer monoid?

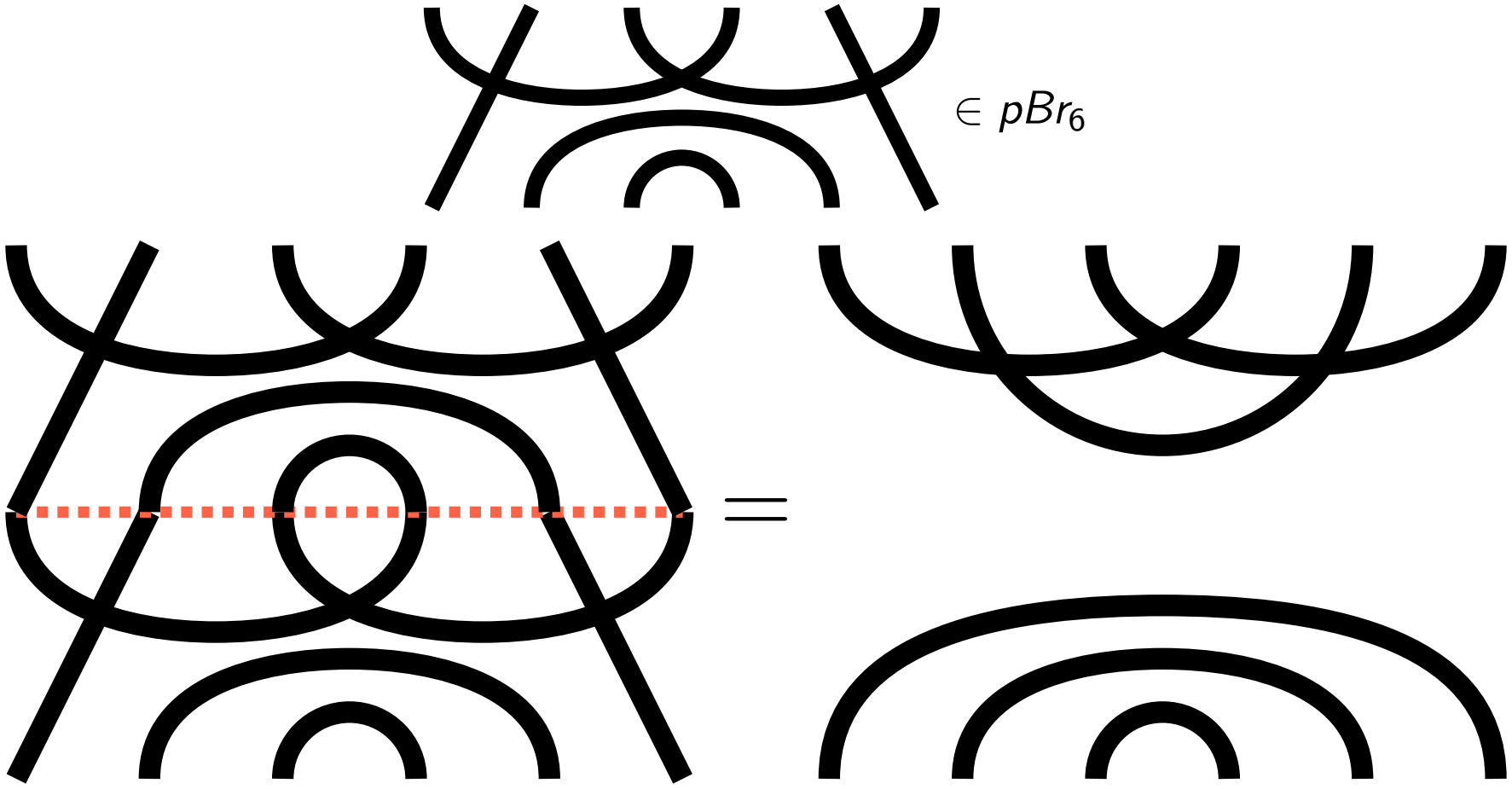
Or: 1d cobordisms!

1d cobordisms



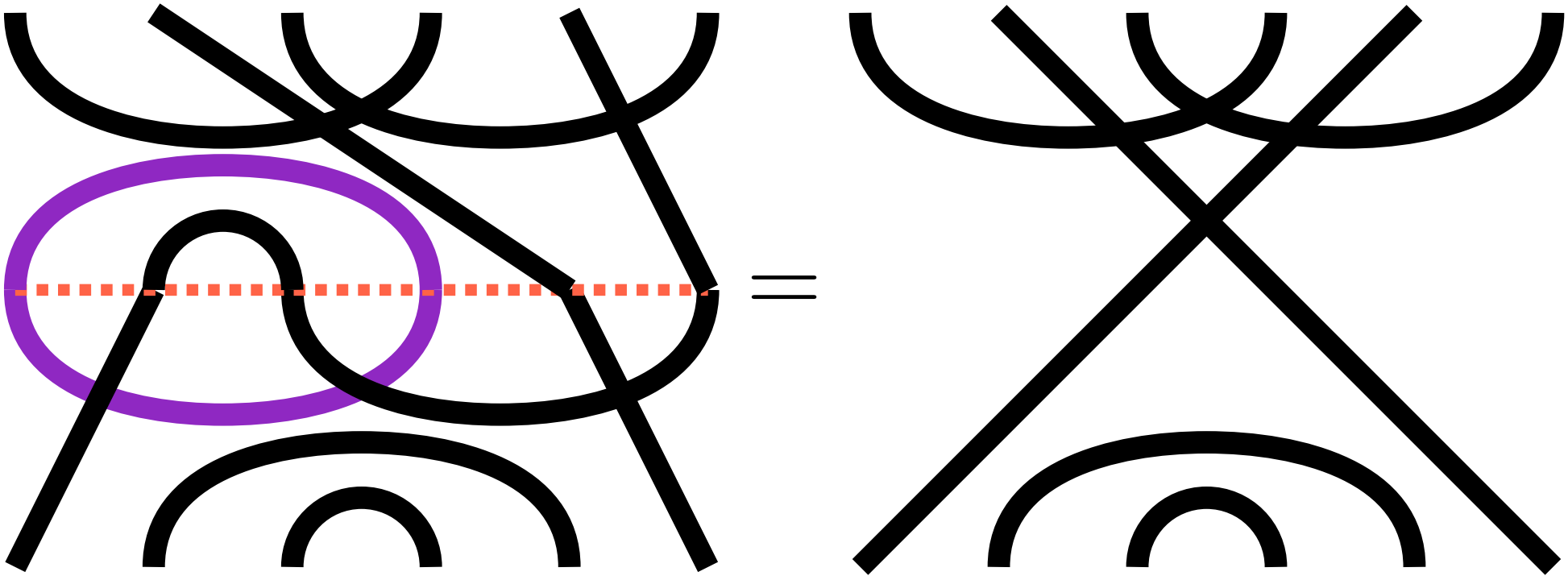
- The collection 1COB "Category" contains points (0-manifolds) "Objects"
- It also contains lines (1-manifolds) "Arrows"
- We can compose lines in a nice way

The Brauer monoid – almost

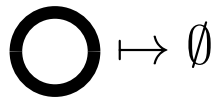


- The pre Brauer monoid pBr_n on n strands is $\mathrm{End}_{1COB}(n)$
- In other words, 1d cobordisms diagrams from n to n points
- Multiplication is stacking

The Brauer monoid

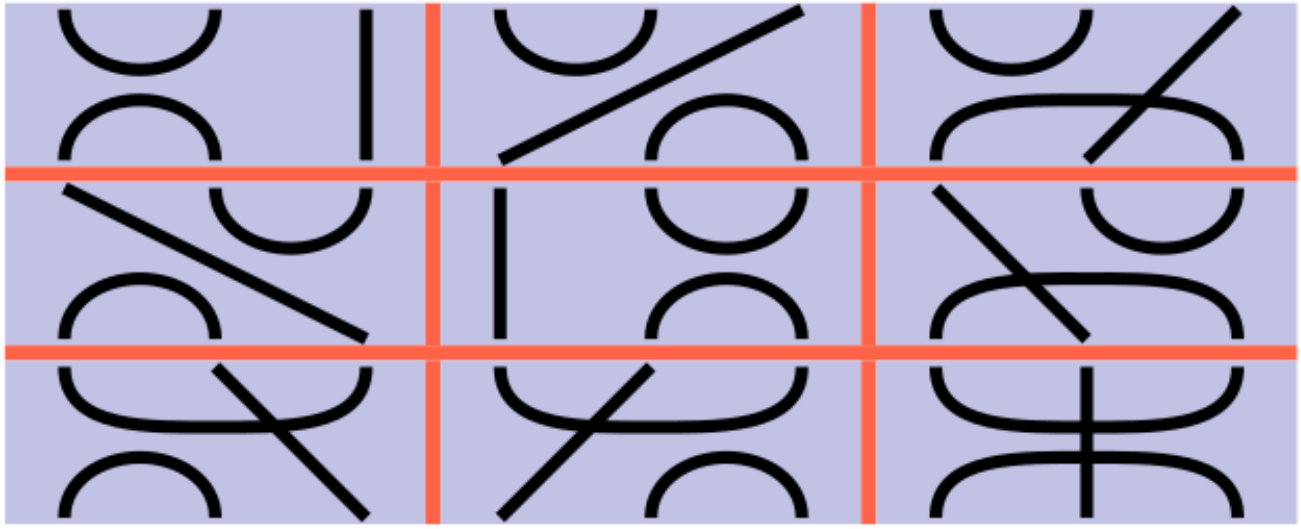


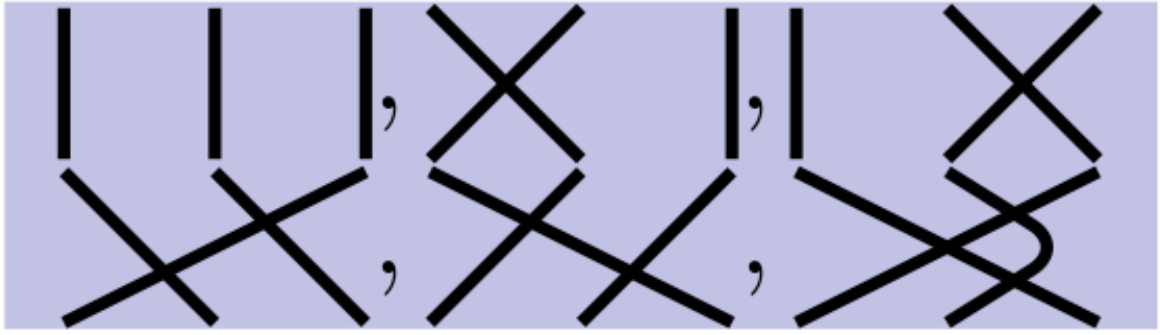
- The Brauer monoid Br_n is pBr_n without circles
- In other words, 1d cobordisms diagrams from n to n points without circles
- Multiplication is stacking and circle removal



For completeness: A formal statement

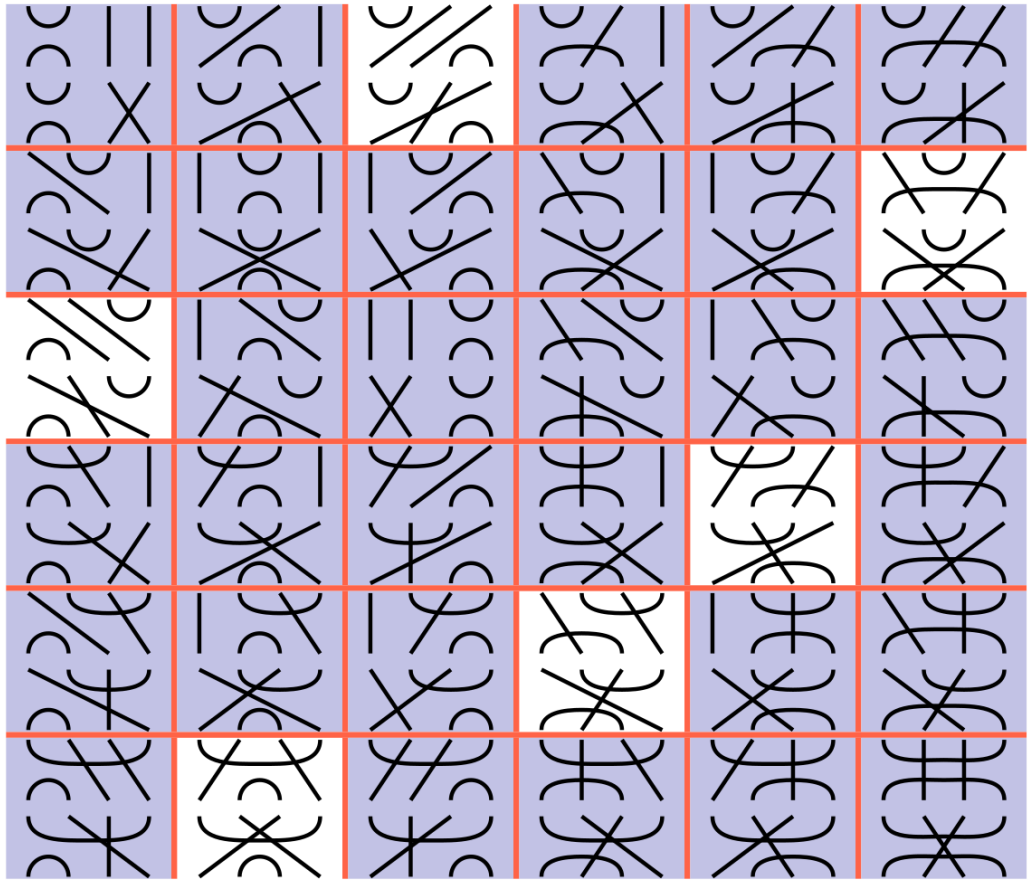
Br_n is a finite monoid with $(2n-1)!! = (2n-1)(2n-3)...1$ elements





Cells in the Brauer monoid

$\mathcal{J}_2$:



- J-cells in Br_n are indexed by through strands,
- Left cells are indexed by bottom and right cells by top diagrams
- Idempotent H-cells are symmetric groups
- H-reduction $\Rightarrow$ nice classification of simple Br_n-reps

Thank you for your attention!

I hope that was of some help.