

What are...examples of character tables?

Or: The ATLAS of finite groups and co

Characters of abelian groups

Character table of C_8

C_8 : Cyclic group

class	1	2	4A	4B	8A	8B	8C	8D	
size	1	1	1	1	1	1	1	1	
ρ_1	1	1	1	1	1	1	1	1	trivial
ρ_2	1	1	1	1	-1	-1	-1	-1	linear of order 2
ρ_3	1	-1	i	$-i$	ζ_8^7	ζ_8^5	ζ_8^3	ζ_8	linear of order 8 faithful
ρ_4	1	1	-1	-1	$-i$	i	$-i$	i	linear of order 4
ρ_5	1	-1	$-i$	i	ζ_8^5	ζ_8^7	ζ_8	ζ_8^3	linear of order 8 faithful
ρ_6	1	-1	i	$-i$	ζ_8^3	ζ_8	ζ_8^7	ζ_8^5	linear of order 8 faithful
ρ_7	1	1	-1	-1	i	$-i$	i	$-i$	linear of order 4
ρ_8	1	-1	$-i$	i	ζ_8	ζ_8^3	ζ_8^5	ζ_8^7	linear of order 8 faithful

- ▶ $\mathbb{Z}/n\mathbb{Z}$ has n simple reps with characters given by n th roots of unities
- ▶ General abelian groups have similar char tables
- ▶ However, beware dividing characteristic

Character table of S_4

S_4 : Symmetric group on 4 letters; = $\text{PGL}_2(\mathbb{F}_3) = \text{Aut}(Q_8) = \text{Hol}(C_2^2)$
symmetries = cube/octahedron rotations

class	1	2A	2B	3	4	
size	1	3	6	8	6	
ρ_1	1	1	1	1	1	trivial
ρ_2	1	1	-1	1	-1	linear of order 2
ρ_3	2	2	0	-1	0	orthogonal lifted from S_3
ρ_4	3	-1	-1	0	1	orthogonal faithful
ρ_5	3	-1	1	0	-1	orthogonal faithful

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- ▶ Reps of symmetric groups is one of the main topics in classical rep theory
 - ▶ The char table of symmetric groups is integral
 - ▶ I will cover them more carefully in other videos

Characters do not determine groups

Character table of D_4

D_4 : Dihedral group; = $\text{He}_2 = \text{A}\Sigma\text{L}_1(\mathbb{F}_4) = 2_+^{1+2}$ = square symmetries

class	1	2A	2B	2C	4	
size	1	1	2	2	2	
ρ_1	1	1	1	1	1	trivial
ρ_2	1	1	-1	1	-1	linear of order 2
ρ_3	1	1	1	-1	-1	linear of order 2
ρ_4	1	1	-1	-1	1	linear of order 2
ρ_5	2	-2	0	0	0	orthogonal faithful

Character table of Q_8

Q_8 : Quaternion group; = $C_4.C_2 = \text{Dic}_2 = 2_-^{1+2}$

class	1	2	4A	4B	4C	
size	1	1	2	2	2	
ρ_1	1	1	1	1	1	trivial
ρ_2	1	1	-1	1	-1	linear of order 2
ρ_3	1	1	1	-1	-1	linear of order 2
ρ_4	1	1	-1	-1	1	linear of order 2
ρ_5	2	-2	0	0	0	symplectic faithful, Schur index 2

- ▶ The char tables of D_4 and Q_8 are the same!
- ▶ One needs more information than char tables to recover the groups

For completeness: A list

Here is a list of important char tables

- ▶ Char tables of abelian groups
 - ▷ Determined by roots of unity
 - ▷ G of order n has n simple reps
- ▶ Char tables of dihedral groups
 - ▷ 2 or 4 1-dim simple reps + some 2-dim simple reps
- ▶ Char tables of symmetric groups
 - ▷ Integral!
- ▶ Char tables of alternating groups
 - ▷ One 1-dim simple rep
 - ▷ each row/column contains at most 2 irrational numbers
- ▶ Char tables of special linear groups
 - ▷ Tend to have “big” nontrivial simple reps
- ▶ Many more!

ATLAS
OF
FINITE GROUPS

Maximal Subgroups and Ordinary
Characters for Simple Groups

BY

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with computational assistance from

J. G. THACKRAY

- ▶ The **ATLAS** lists the char tables (and even more) of 93 finite simple groups
- ▶ Serre: “can’t think of any other book published in the last 50 years which had such an impact”

Thank you for your attention!

I hope that was of some help.