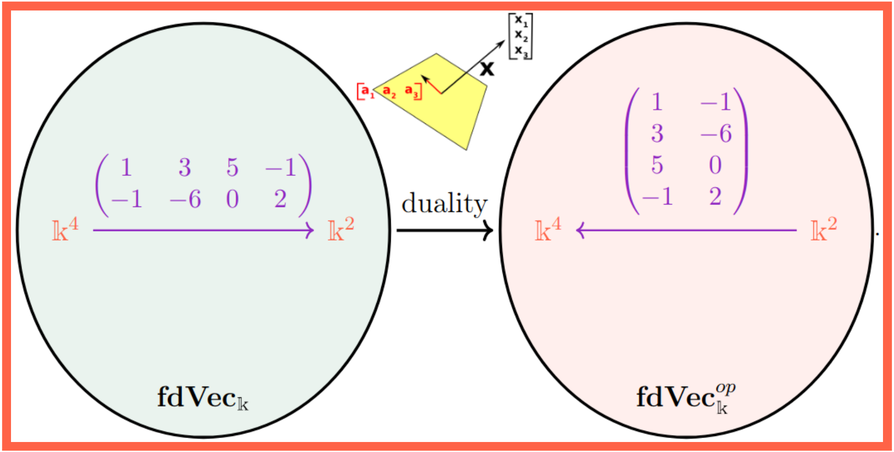
What is...quantum topology - part 6?

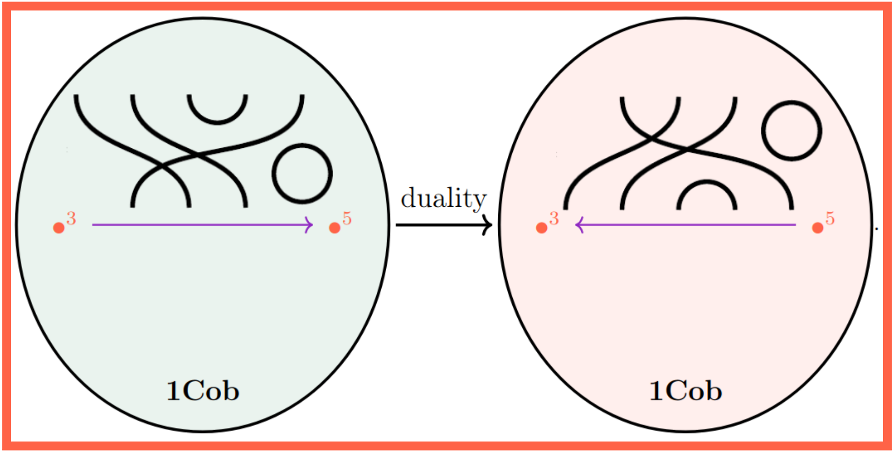
Or: Categories 4 from Chapter 1

Transposing matrices



- In $\mathrm{fdVec}_{\mathbb{k}}$ (fin. dim. vector spaces) duality is taking the dual vector space
- Duality on objects does not change the object $(\mathbb{k}^n)^* \cong \mathbb{k}^n$ Fix
- Duality transposed matrices and reverses their direction Flip

Flipping diagrams



- In 1Cob or 1Tan duality is taking a mirror along $y = 0$
- Duality on objects does not change the object $n^* = n$ Fix
- Duality flips diagrams and reverses their direction Flip

Was it a car or a cat I saw?

Statements have dual/co statements, *e.g.*:

- $P_C(X)$ $\forall$ Y in C $\exists!$ $f: X \to Y$ in C
- $P_{C^{op}}(X)$ $\forall$ Y in C^{op} $\exists!$ $f: X \to Y$ in C^{op}
- $P_C^{op}(X)$ $\forall$ Y in C $\exists!$ $f: X \leftarrow Y$ in C

For completeness: A formal definition

The opposite category C^{op} of C has:

(a) The same same objects

(b) An opposite morphism $f^{op}\colon Y \to X$ for each $f\colon X \to Y$ in C

(c) Composition $f^{op}g^{op} = (gf)^{op}$

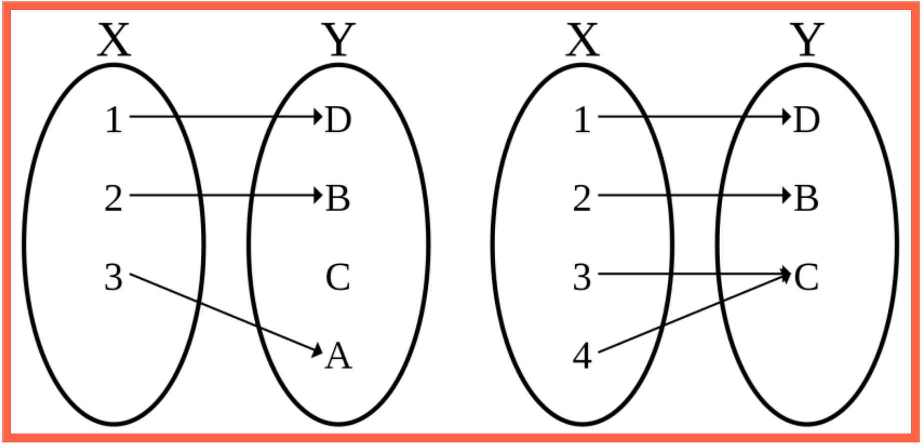
- Duality Principle

  Property P holds $\forall$ categories $\Leftrightarrow$ property P^{op} holds $\forall$ categories

  $(C^{op})^{op} = C$, and P^{op}_C holds if and only if $P_{C^{op}}$ holds

- In general, $C \ncong C^{op}$ and $P_C \neq P^{op}_C$ but categories and properties can be self-dual, *e.g.*:
  - $\mathrm{fdVec}_{\mathbb{k}}$ and 1Cob are self-dual
  - "Being an identity arrow" is self-dual

Duality in action



(a) $P_C(f)$ $\exists\, g\colon Y \to X$ with $X \xrightarrow{f} Y \xrightarrow{g} X = id_X$

(b) $P_C^{op}(f)$ $\exists\, g\colon Y \leftarrow X$ with $X \xleftarrow{f} Y \xleftarrow{g} X = id_X$

- (a) holds in SET if and only if f is injective (or $f = id_\emptyset$ for $X = \emptyset$)
- (b) holds in SET if and only if f is surjective

Thank you for your attention!

I hope that was of some help.