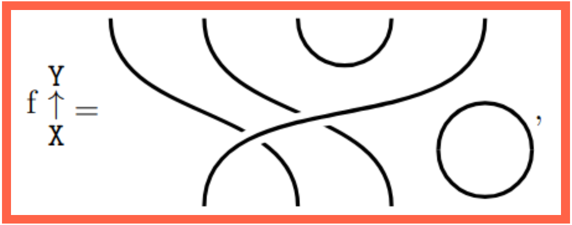
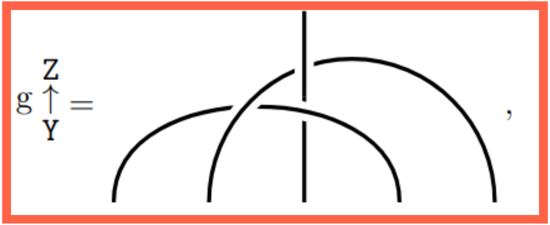
What is...quantum topology - part 4?

Or: Categories 2 from Chapter 1

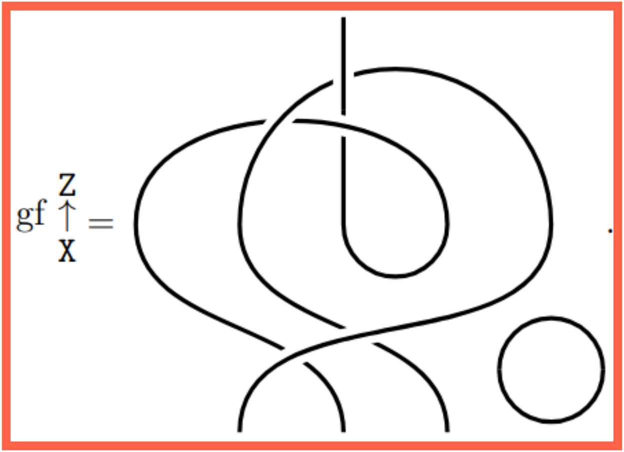
A main example





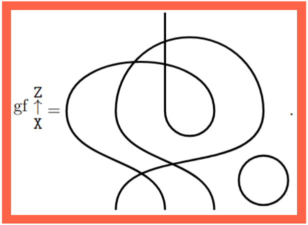
- Above The category of tangles 1Tan
- Objects are boundary points $\bullet$, $\bullet^2 = \bullet\bullet$ etc,
- Morphisms are tangles, as above, with $X = \bullet^3$, $Y = \bullet^5$ and $Z = \bullet$

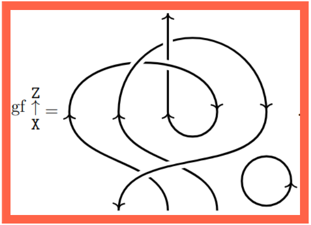
Composition



- Recall A category's main structure is composition
- Composition in 1Tan $gf = g \circ f =$ "stack g on f"
- Above The composition of f and g from the previous slide

Variations





- Top The category 1Cob of cobordism (= 1Tan without embedding)
- Bottom The category 1State of states (= 1Tan with orientation)
- Above The composition of f and g from the previous slide

For completeness: A formal definition

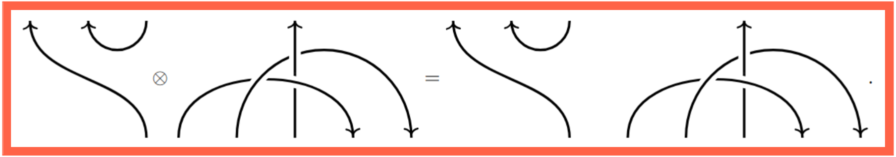
The category 1Tan of 1 dimensional tangles is defined:

- Its objects are 0 dimensional manifolds $\bullet^n = \bullet ... \bullet$ for $n \in \mathbb{N}$ a.k.a. points
- Its morphisms are 1 dimensional cobordisms between these a.k.a. strands
- The strands are not oriented but embedded
- Composition = stacking

Variations:

- 1Cob (no embedding)
- 1State (embedding + orientations)

There is also a monoidal structure discussed later; its given by juxtaposition:



Quantum topology without topology

	monoidal	braided	pivotal	symmetric	self-dual •	Reidemeister 1	topology
Br	**Y**	**Y**	**Y**	Y	Y	Y	**1Cob**
qBr	**Y**	**Y**	**Y**	N	Y	Y	**1Tan**
oqBr	**Y**	**Y**	**Y**	N	N	Y	**1State**
orqBr	**Y**	**Y**	**Y**	N	N	N	**1Ribbon**

A ***quantum invariant*** Q is a structure preserving functor

$$Q\colon \_\mathbf{Br} \to \mathbf{C},$$

where **C** is "a linear algebra like category".

- Main goal Introduce these categories rigorously
- Different names The algebraic incarnations are called Brauer categories ??Br
- Quantum invariants are invariants of such categories

Thank you for your attention!

I hope that was of some help.