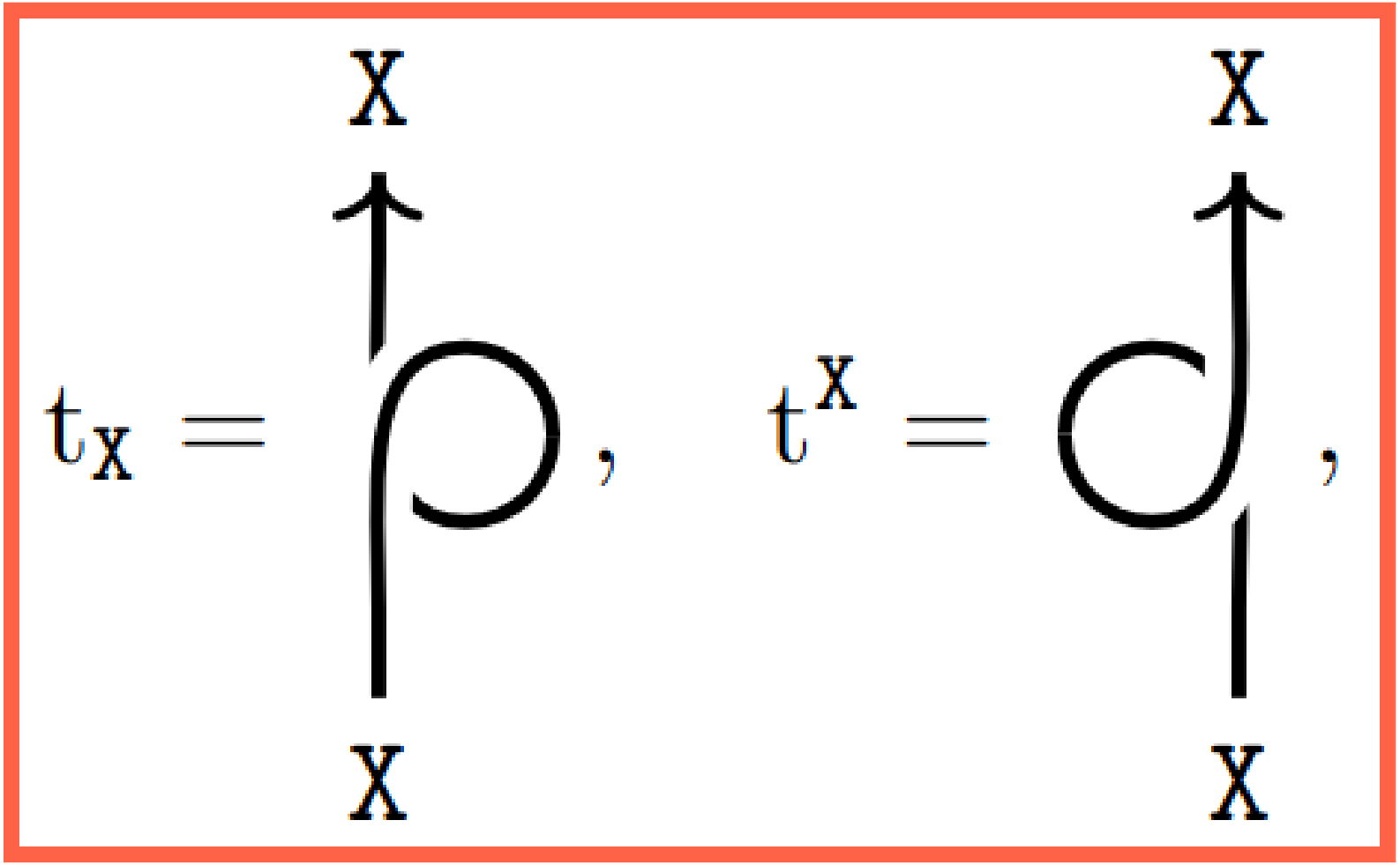
What is...quantum topology - part 26?

Or: Braided categories 5 from Chapter 5

From braidings to twists

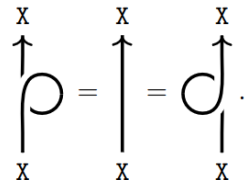


- In a braided pivotal category, every object has right and left twists
- Diagrammatically, a twist is a curl built from braiding, cups and caps
- So braiding + duality naturally produces the Reidemeister 1 type data

Three versions of Reidemeister 1

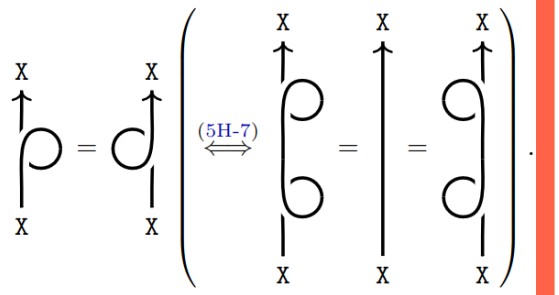
First, the ***(classical) Reidemeister*** 1 ***moves*** are

(5H-2)



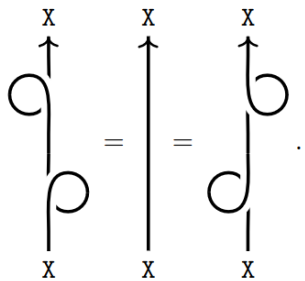
Second, the ***ribbon equation*** is

(5H-3)



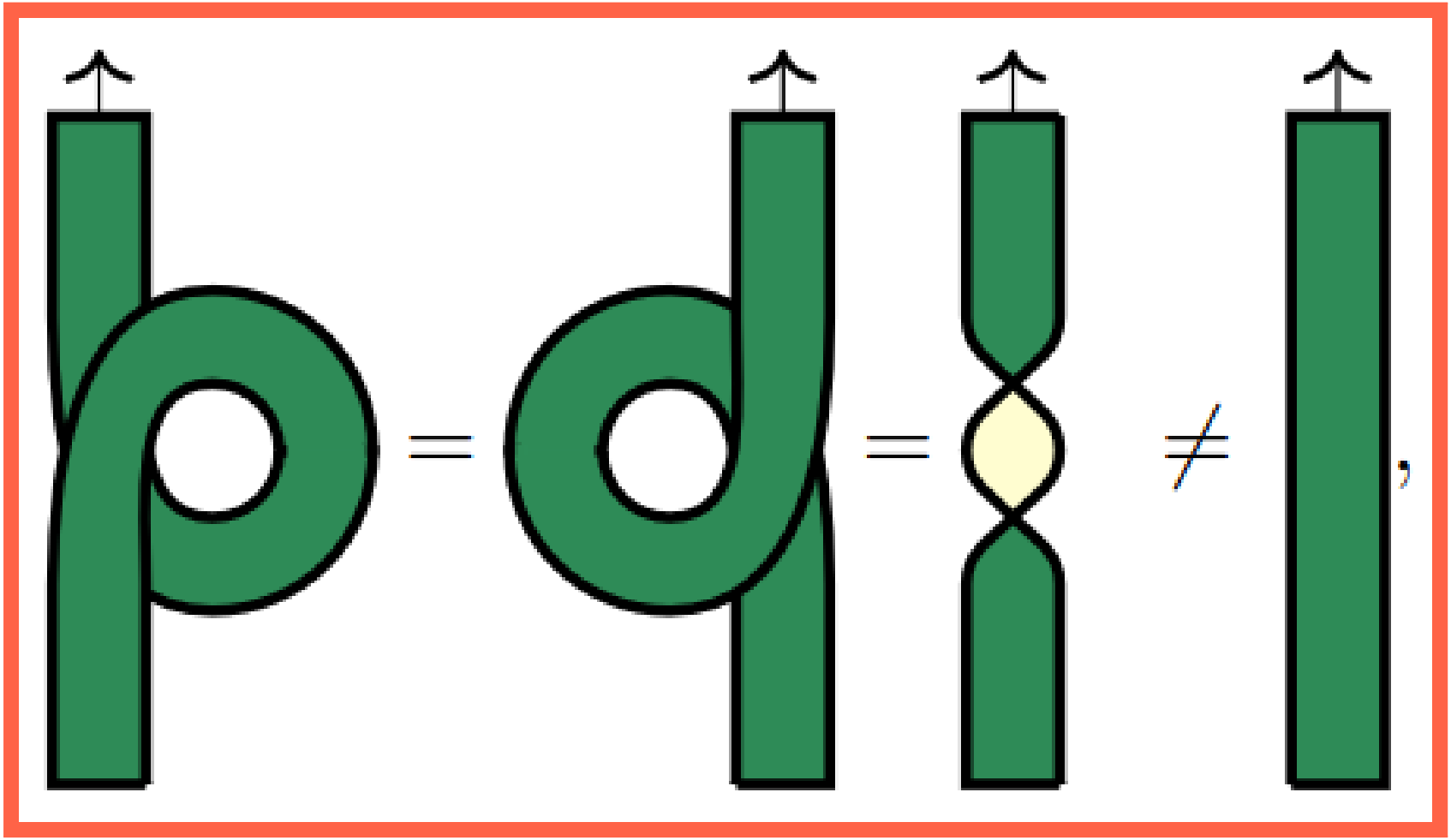
Finally, the ***framed Reidemeister*** 1 ***moves*** are

(5H-4)



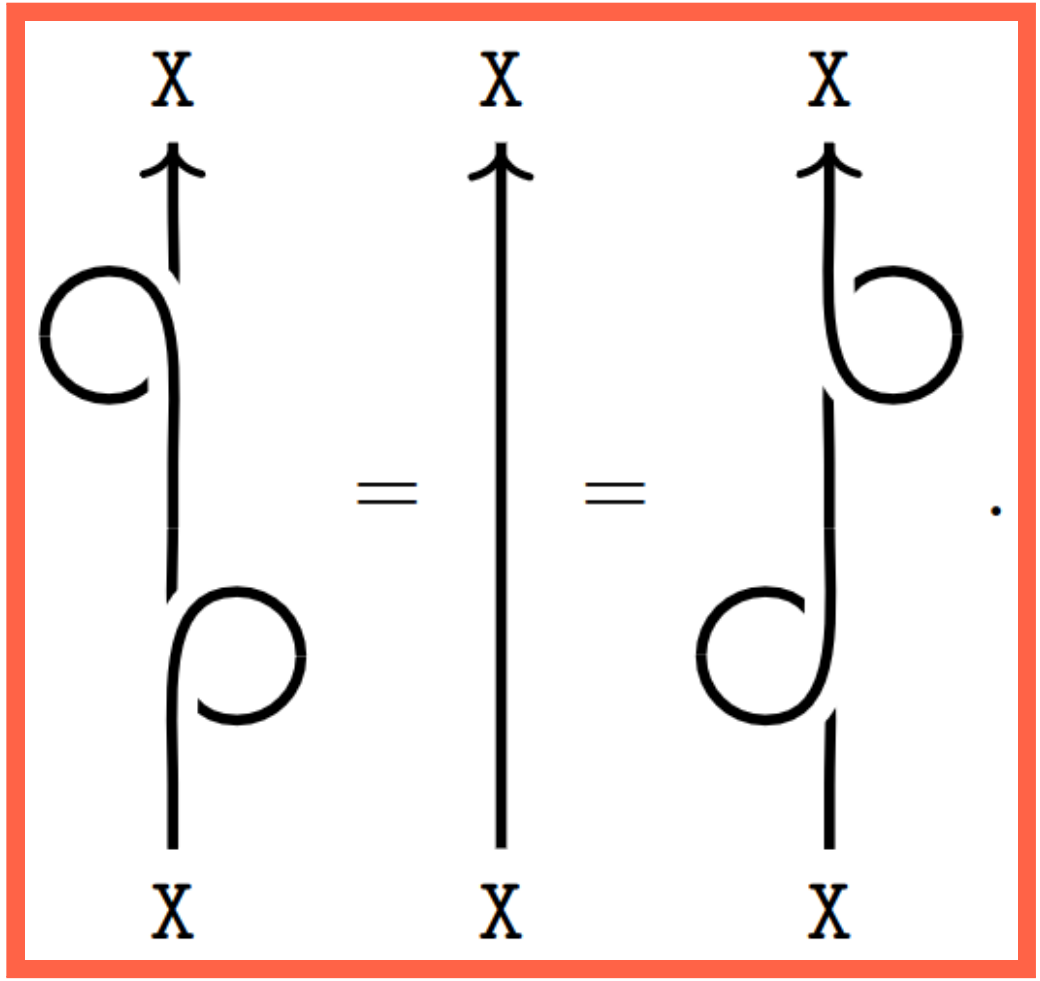
- The classical Reidemeister 1 move says a curl simply disappears
- The ribbon equation identifies the left and right twists
- The framed Reidemeister 1 move replaces a curl by the corresponding twist

Why these notions differ



- For vector spaces the classical Reidemeister 1 holds
- But sometimes e.g. ribbon holds but classical Reidemeister 1 fails
- So the distinction between braided, framed and ribbon is genuinely real

What always holds



- In any braided pivotal category, the framed Reidemeister 1 relation holds
- This comes from the naturality and sliding properties of the twists
- Hence twists are not a gimmick but built into the braided pivotal calculus

Ribbon categories

- A ribbon category is a braided pivotal category satisfying the ribbon equation
- This is the right algebraic setting for knot invariants
- So: ribbon structure is where the topology comes back in

Thank you for your attention!