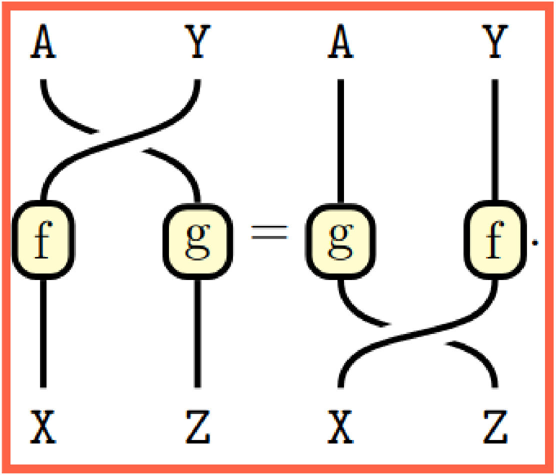
What is...quantum topology - part 24?

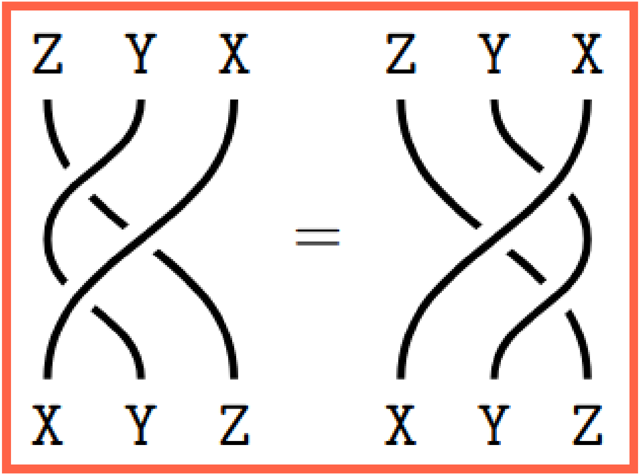
Or: Braided categories 3 from Chapter 5

The braid relation = Yang–Baxter



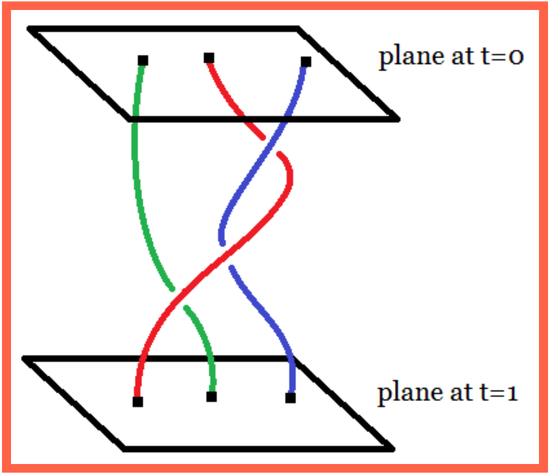
- A braid is built from crossings, but it must satisfy the braid relation
- On three neighboring strands this says $\sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1}$
- In a braided category this equality is forced by the hexagon coherence

What is the braid group B_n?



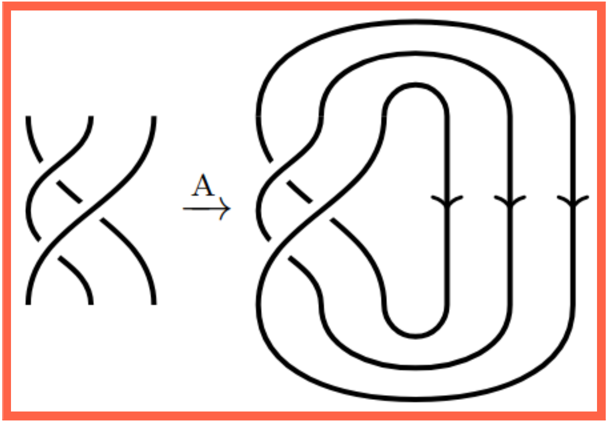
- The braid group B_n consists of n-strand braids up to deformation fixing endpoints
- It is generated by $\sigma_1, \ldots, \sigma_{n-1}$, where σ_i crosses strands i and $i+1$
- Relations Far-comm. $\sigma_i \sigma_j = \sigma_j \sigma_i$ for $|i - j| \geq 2$ plus the braid relation above

Braid group action on tensor powers



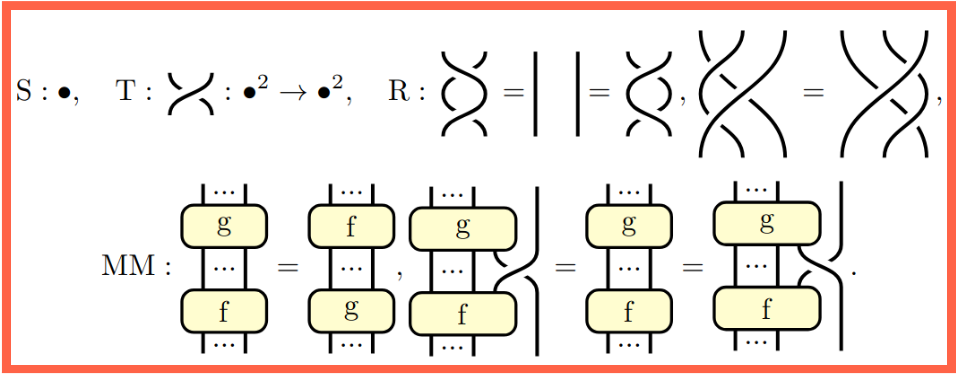
- A braided category gives a braid group action on $X^{\otimes n}$
- Generator σ_i acts by inserting $\beta_{X,X}$ in the ith position
- So we get representations $B_n \to \mathrm{Aut}(X^{\otimes n})$ for all n

Closing a braid: from braids to links



- Closing connects endpoints to turn a braid into a link diagram
- Categorically we want a number from an endomorphism of $X^{\otimes n}$
- That uses caps/cups and a trace-like closure on diagrams

Markov moves and invariants



- Different braids can have the same closure
- Closures agree exactly up to Markov moves
- A link invariant comes from a braid action plus a Markov trace

Thank you for your attention!