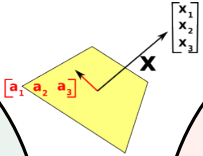
What is...quantum topology - part 19?

Or: Pivotal categories 1 from Chapter 4

Duality: transposing matrices



$$\mathbb{K}^4 \xrightarrow{\begin{pmatrix} 1 & 3 & 5 & -1 \\ -1 & -6 & 0 & 2 \end{pmatrix}} \mathbb{K}^2$$

$\mathbf{fdVec}_{\mathbb{K}}$

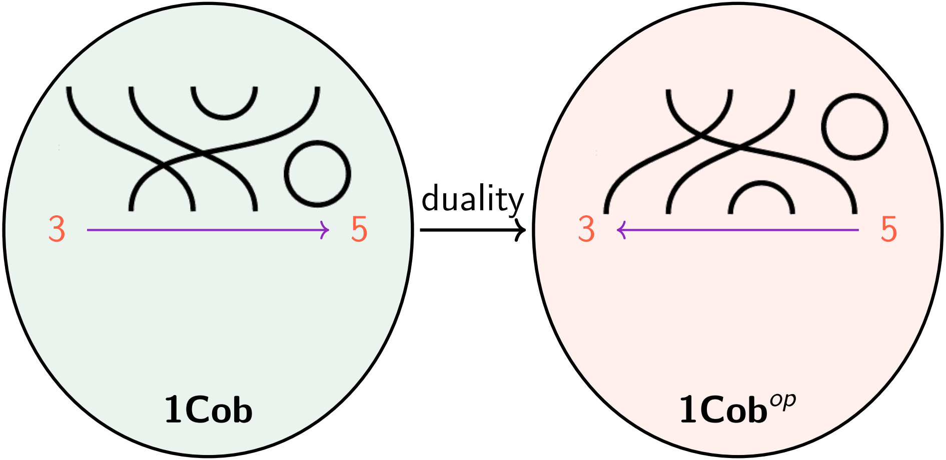
duality $\longrightarrow$

$$\mathbb{K}^4 \xleftarrow{\begin{pmatrix} 1 & -1 \\ 3 & -6 \\ 5 & 0 \\ -1 & 2 \end{pmatrix}} \mathbb{K}^2$$

$\mathbf{fdVec}_{\mathbb{K}}^{op}$

- In $\mathbf{fdVec}_{\mathbb{K}}$ duality is taking the dual vector space
- Duality on objects does not change the object $(\mathbb{K}^n)^* \cong \mathbb{K}^n$ Fix
- Duality transposed matrices and reverses their direction Flip

Duality: flipping diagrams



- In **1Cob** duality is taking a mirror along $y = 0$
- Duality on objects does not change the object $n^* = n$ Fix
- Duality flips diagrams and reverses their direction Flip

Singularities

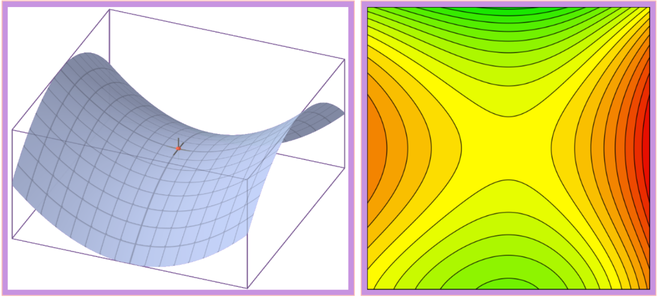


FIGURE 9. In a nutshell, Morse theory is the idea that most point on a manifold have nicely behaved neighborhoods and only a few critical points (called ***Morse points***) where the behavior changes drastically need to be studied. The pictures show a saddle point on a surface and its contour lines; this is a prototypical examples of a critical point where the behavior of a system changes.

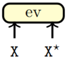
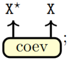
Pictures from https://en.wikipedia.org/wiki/Morse_theory.

- Singularities = points of drastic change
- These are of crucial importance (not just in math)
- Goal Have singularities in diagrams

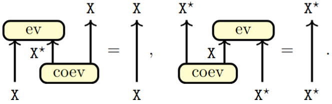
For completeness: A formal definition

Definition 4B.1. A ***right dual*** $(\mathtt{X}^{\star}, \mathrm{ev}_{\mathtt{X}}, \mathrm{coev}_{\mathtt{X}})$ of $\mathtt{X} \in \mathbf{C}$ in a category $\mathbf{C} \in \mathbf{MCat}$ consists of

- an object $\mathtt{X}^{\star} \in \mathbf{C}$;
- a ***(right) evaluation*** $\mathrm{ev}_{\mathtt{X}}$ and a ***(right) coevaluation*** $\mathrm{coev}_{\mathtt{X}}$, *i.e.* morphisms

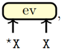
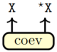
(4B-2) $\mathrm{ev}_{\mathtt{X}} \colon \mathtt{X}\mathtt{X}^{\star} \to \mathbb{1} \rightsquigarrow$ , $\mathrm{coev}_{\mathtt{X}} \colon : \mathbb{1} \to (\mathtt{X}^{\star})\mathtt{X} \rightsquigarrow$ ;

such that they are ***non-degenerate***, *i.e.*

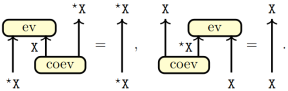
(4B-3) 

Similarly, a ***left dual*** $({}^{\star}\mathtt{X}, \mathrm{ev}^{\mathtt{X}}, \mathrm{coev}^{\mathtt{X}})$ of $\mathtt{X} \in \mathbf{C}$ in a category $\mathbf{C} \in \mathbf{MCat}$ consists of

- an object ${}^{\star}\mathtt{X} \in \mathbf{C}$;
- a ***(left) evaluation*** $\mathrm{ev}^{\mathtt{X}}$ and a ***(left) coevaluation*** $\mathrm{coev}^{\mathtt{X}}$, *i.e.* morphisms

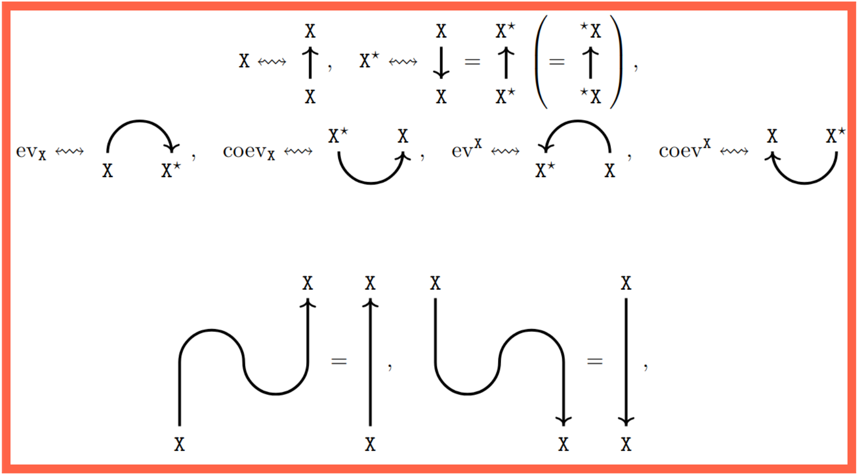
(4B-4) $\mathrm{ev}^{\mathtt{X}} \colon {}^{\star}\mathtt{X}\mathtt{X} \to \mathbb{1} \rightsquigarrow$ , $\mathrm{coev}^{\mathtt{X}} \colon : \mathbb{1} \to \mathtt{X}({}^{\star}\mathtt{X}) \rightsquigarrow$ ;

such that they are ***non-degenerate***, *i.e.*

(4B-5) 

We call (4B-3) and (4B-5) the ***zigzag relations***.

Duality and Morse points



- Idea Use an orientation to distinguish X and its dual
- Evaluations and coevaluations become Morse points
- Non-degeneracy becomes straightening

Thank you for your attention!

I hope that was of some help.