

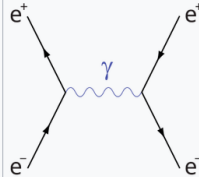
What is...quantum topology - part 18?

Or: Monoidal categories 6 from Chapter 3

Feynman diagrams

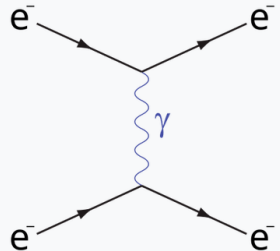
Pair production
and annihilation

In the Stückelberg–Feynman interpretation, pair annihilation
is the same process as pair production



Møller scattering

electron-electron scattering



- ▶ Above Feynman diagrams (originate ≈ 1949)
- ▶ Feynman diagram = pictorial representation of the mathematical expressions
- ▶ Note the appearance of trivalent vertices

Applications of Negative Dimensional Tensors

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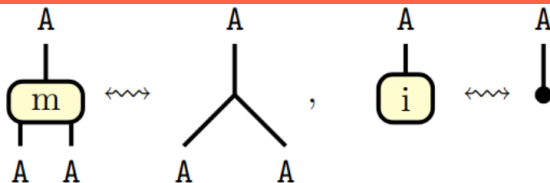
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$$= \theta_c^{ab},$$

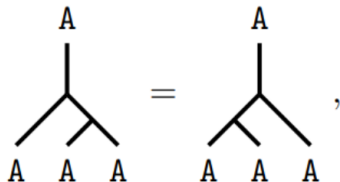

$$= \chi_{bcd}^a.$$

- ▶ Above Penrose diagrams (originate ≈ 1971)
- ▶ Penrose diagram = pictorial representation of the mathematical expressions
- ▶ Note the appearance of trivalent vertices

Trivalent vertices and multiplication



are the structure maps and



- Above Multiplication diagrams (originate ?)
- Multiplication diagram = pictorial representation of the mathematical expressions
- Note the appearance of trivalent vertices

For completeness: A formal definition

Webs

Example 3G.2. The (*generic*) *web category* **Web** (also known as the generic *trivalent* category or *spider* category or generic *birdtracks* category or ...) is defined as follows. We let

$$S : \bullet, \quad T : \cap : \bullet \otimes \bullet \rightarrow \mathbb{1}, \quad \cup : \mathbb{1} \rightarrow \bullet \otimes \bullet, \quad \cap : \bullet \otimes \bullet \otimes \bullet \rightarrow \mathbb{1}, \quad \cup : \mathbb{1} \rightarrow \bullet \otimes \bullet \otimes \bullet,$$

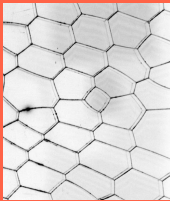
and the generating morphisms are called *bilinear* and *trilinear (co)form*, respectively. Using [Equation 3G-1](#) and similar expressions to define

$$R : \text{[diagram]} = \text{[diagram]}, \quad \text{[diagram]} = \text{[diagram]}, \quad \text{[diagram]} = \text{[diagram]},$$

called *isotopies* (or *non-degeneracy conditions*).



Example



Group graded vector spaces diagrammatically

$$S : G, \quad T : \left\{ \begin{array}{l} gh \\ \text{diagram} : g \otimes h \rightarrow gh, \\ g \quad h \end{array} \right. \quad \begin{array}{l} \text{cap} : g \otimes g^{-1} \rightarrow \mathbb{1}, \quad \text{cup} : \mathbb{1} \rightarrow g \otimes g^{-1}, \\ \text{cap} : g^{-1} \otimes g \rightarrow \mathbb{1}, \quad \text{cup} : \mathbb{1} \rightarrow g^{-1} \otimes g. \end{array}$$

$$R : \left\{ \begin{array}{l} \text{associativity}, \quad \text{commutativity}, \quad \text{associativity}, \\ \text{empty set}, \quad \text{multiplication}, \quad \text{comultiplication}. \end{array} \right.$$

- **Theorem** The category of G -graded vector spaces $\mathbf{Vec}_{\mathbb{C}}(G)$ has the above diagrammatic presentation
- This is a **quotient** of the category of webs
- **Proof?** E.g. find very consequences of the relations

$$(\text{cap} \mid) \circ (\mid \mid \text{cup}) = \text{diagram} \quad \text{is isotopic to} \quad \text{diagram}$$

Thank you for your attention!

I hope that was of some help.