What is...quantum topology - part 16?

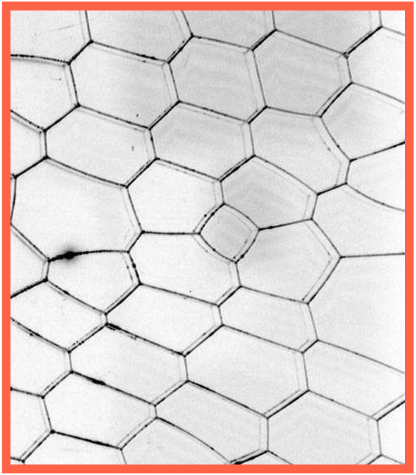
Or: Monoidal categories 4 from Chapter 2

Super mathematics

+	$[0]_2$	$[1]_2$
$[0]_2$	$[0]_2$	$[1]_2$
$[1]_2$	$[1]_2$	$[0]_2$

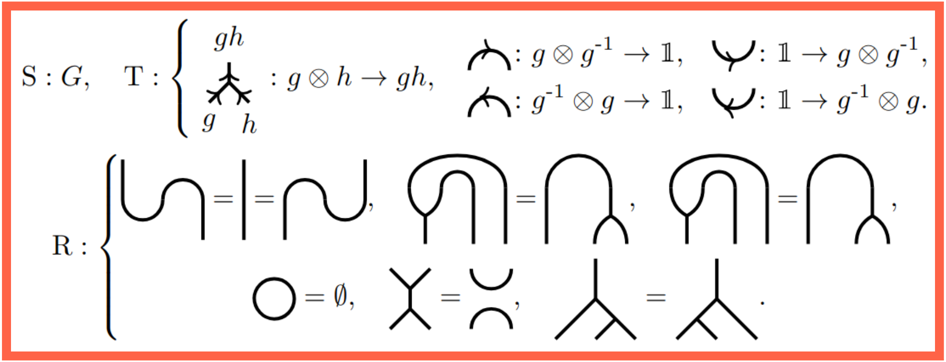
- **Super** $= \mathbb{Z}/2\mathbb{Z}$ graded (think: even and odd; bosons and fermions)
- **Monoidal rules** even$\otimes$even$=$odd$\otimes$odd$=$even, even$\otimes$odd$=$odd$\otimes$even$=$odd
- **This video** The category $Vec_{\mathbb{C}}^{dia}(\mathbb{Z}/2\mathbb{Z})$ of super vector spaces diagrammatically

Webs



- Webs = trivalent graphs (potentially with boundary i.e. degree one vertices)
- Above A web (a cut through a soap foam)
- These are morphisms in $\mathit{Vec}^{dia}_{\mathbb{C}}(\mathbb{Z}/2\mathbb{Z})$

Super vector spaces $Vec^{dia}_{\mathbb{C}}(\mathbb{Z}/2\mathbb{Z})$

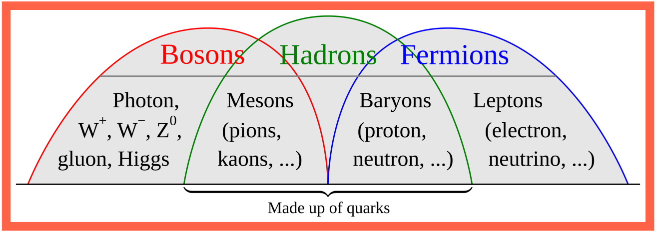


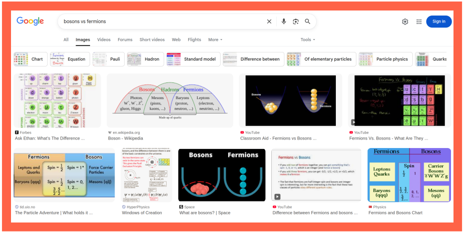
- Objects = boundary points = elements of $\mathbb{Z}/2\mathbb{Z}$ (not drawing the trivial element $\mathbb{1}$)
- Morphisms = webs modulo relations as above
- This actually works for any grading group G where the objects are now elements of G and morphisms are labeled webs

For completeness: A formal definition

Upon taking direct and allowing scalars, $Vec_{\mathbb{C}}^{dia}(\mathbb{Z}/2\mathbb{Z})$ is (monoidally equivalent to) the category of super vector spaces $Vec_{\mathbb{C}}(\mathbb{Z}/2\mathbb{Z})$

These come from a mathematical theory describing bosons versus fermions





Generators-relations

Groups encoded efficiently

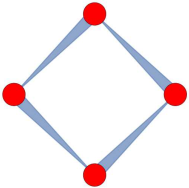
$\mathbb{Z}/4\mathbb{Z}$ (written additively):

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

1 is a generator of $\mathbb{Z}/4\mathbb{Z}$:

$\emptyset = 0, \quad 1 = 1, \quad 11 = 1+1 = 2, \quad 111 = 1+1+1 = 3$

Illustrated as a graph:



- Above A group given as a multiplication table; like $Vec_{\mathbb{C}}(\mathbb{Z}/2\mathbb{Z})$
- Above 2 A group given by generators-relations; like $Vec_{\mathbb{C}}^{dia}(\mathbb{Z}/2\mathbb{Z})$
- Game we will play Try to find generators-relations of 'nice' categories

Thank you for your attention!

I hope that was of some help.