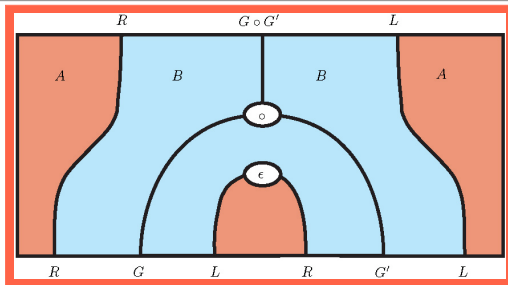


**What is...quantum topology - part 12?**

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Or: Categories 10 from Chapter 1

# Two-dimensional algebra



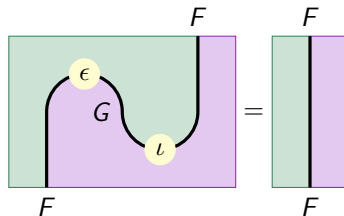
► Functor calculus gets a 2d flavor via string diagrams

► Important Do not draw identity functors  $id_C$ , e.g.

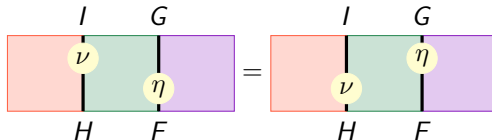
$$\begin{array}{l}
 F: C \rightarrow D, G: D \rightarrow C \\
 \epsilon: FG \Rightarrow id_D
 \end{array}
 \iff
 \begin{array}{c}
 \text{Diagram 1: A green rectangle with a purple semi-circle at the bottom labeled } F \text{ and } G. \text{ Inside the semi-circle is a yellow circle labeled } \epsilon. \\
 \text{Diagram 2: A green rectangle with a purple semi-circle at the bottom labeled } F \text{ and } G. \text{ Inside the semi-circle is a yellow circle labeled } \epsilon. \text{ A vertical line labeled } id_D \text{ extends from the top of the semi-circle to the top boundary.}
 \end{array}
 =
 \begin{array}{c}
 \text{Diagram 2}
 \end{array}$$

► Goal Rediscover adjoint functors using plane geometry only

# Zigzag



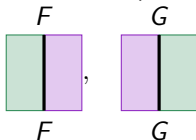
- There are several compatibility conditions we need anyway , e.g.



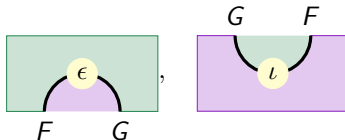
- The zigzag relation is a genuine and crucial relation planar diagrams satisfy
- Use this to define certain functors

# What do we need for the zigzag?

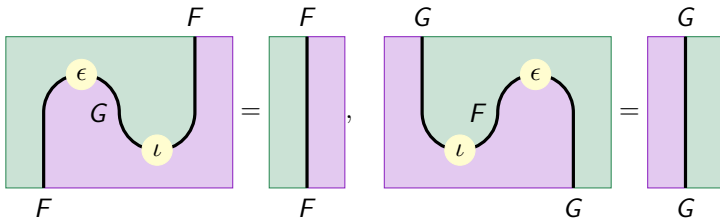
- We need functors ( $F: C \rightarrow D, G: D \rightarrow C$ ) in opposite directions



- We need nat trafos  $\epsilon: FG \Rightarrow id_D$  and  $\iota: id_C \Rightarrow GF$



- We need the relations



## For completeness: A formal definition

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Two functors  $(F, G) = (F: C \rightarrow D, G: D \rightarrow C)$  for an adjoint pair if:

- There exists a counit  $\epsilon: FG \Rightarrow id_D$
- There exists a unit  $\iota: id_C \Rightarrow GF$
- They satisfy the zigzag relations:

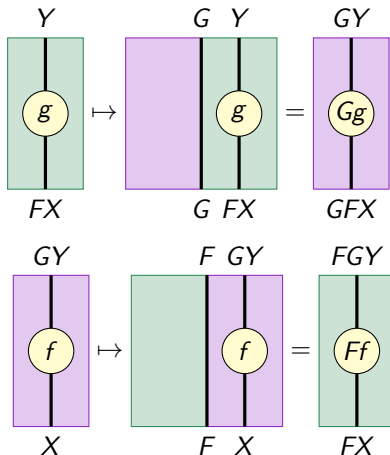
$$\begin{array}{ccc} F & \xrightarrow{id_F \otimes \iota} FG & \xrightarrow{\epsilon \otimes id_F} F \\ & \searrow & \nearrow \\ & id_F & \end{array}$$
$$\begin{array}{ccc} G & \xrightarrow{\iota \otimes id_G} GFG & \xrightarrow{id_G \otimes \epsilon} G \\ & \searrow & \nearrow \\ & id_G & \end{array}$$

In this case  $F$  is the left adjoint of  $G$ , and  $G$  is the right adjoint of  $F$

---

- A functor might not have left/right adjoints
- If they exist, then they are unique up to unique isomorphism

## The hom adjunction



- For  $(F, G)$  we have  $\text{hom}_D(FX, Y) \cong \text{hom}_C(X, GY)$
- How can we see this? Use the diagrams above!

**Thank you for your attention!**

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I hope that was of some help.