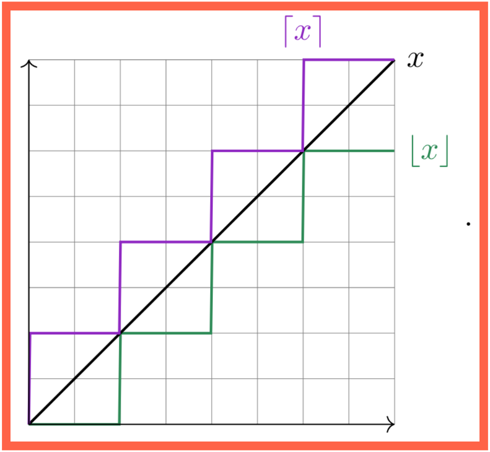
What is...quantum topology - part 11?

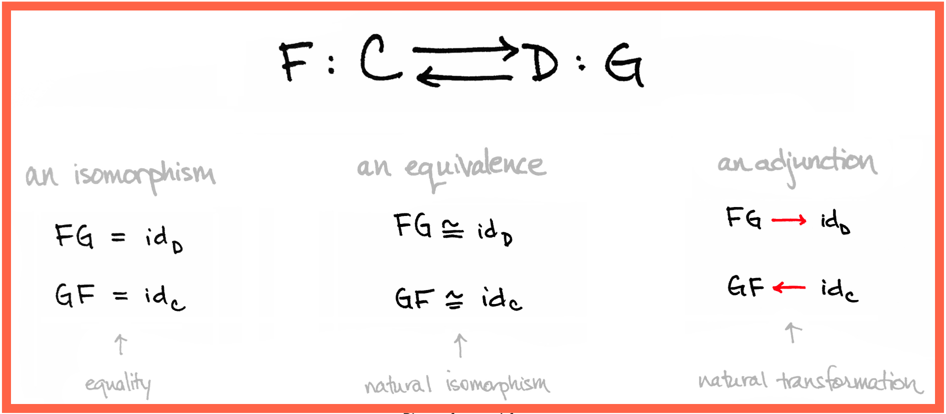
Or: Categories 9 from Chapter 1

Life is not invertible



- The inclusion $\iota \colon \mathbb{Z} \to \mathbb{R}$ is not invertible and $\mathbb{Z} \ncong \mathbb{R}$
- There is no invertible way to assign an integer to a real number
- Ceiling and floor serve as approximations of inverses

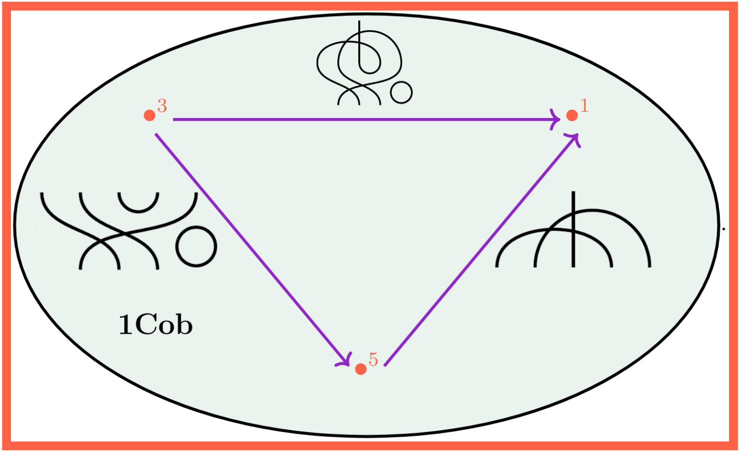
No objects, please!



Picture from ... I forgot

- Equality $=$ is the "wrong" notion in category theory
- Equivalence $\cong$ is much better but still involves objects
- Idea Weaken the condition $\cong$ by ignoring objects

The free categories



- Free functor = pseudo-inverse of a forgetful functor
- Example 1 Free vector space Set $\to$ $\mathrm{Vec}_{\Bbbk}$
- Example 2 1Cob is a free pivotal symmetric category (defined later)

For completeness: A formal definition

Two functors $(F, G) = (F\colon C \to D, G\colon D \to C)$ for an adjoint pair if:

- There exists a nat trafo $\alpha\colon \hom_D(F\_, \_) \Rightarrow \hom_C(\_, G\_)$ (part of the data)
- For all X, Y there are isomorphism3

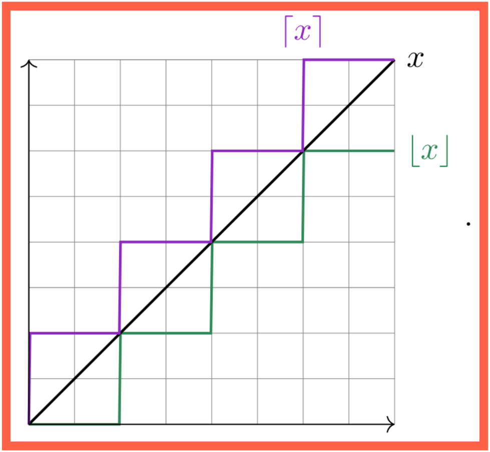
$$\alpha_{X,Y}\colon \hom_D(FX, Y) \xrightarrow{\cong} \hom_C(X, GY)$$

In this case F is the left adjoint of G, and G is the right adjoint of F

- A functor might not have left/right adjoints
- If they exist, then they are unique up to unique isomorphism

> The slogan is "Adjoint functors arise everywhere".
>
> — Saunders Mac Lane, *Categories for the Working Mathematician*

Back to ceiling and floor



- Take $\mathbb{R}$ with $x \to y$ if $x \leq y$, $\mathbb{Z} \subset \mathbb{R}$, $\iota\colon \mathbb{Z} \to \mathbb{R}$ inclusion
- Adjoint functors $(\lceil \_ \rceil, \iota)$ and $(\iota, \lfloor \_ \rfloor)$

$$\lceil y \rceil \leq x \Leftrightarrow y \leq x \quad x \leq \lfloor y \rfloor \Leftrightarrow x \leq y \qquad x \in \mathbb{Z}, y \in \mathbb{R}$$

Thank you for your attention!

I hope that was of some help.