What is...quantum topology - part 1?

Or: Introduction

What is quantum topology (QT)? A Rosetta stone!

Category theory	Algebra	Topology	Physics	Logic
objects $\mathtt{X}$	algebraic data $\mathtt{X}$	manifold $\mathtt{X}$	system $\mathtt{X}$	proposition $\mathtt{X}$
morphism $f\colon \mathtt{X} \to \mathtt{Y}$	relation $f\colon \mathtt{X} \to \mathtt{Y}$	cobordism $f\colon \mathtt{X} \to \mathtt{Y}$	process $f\colon \mathtt{X} \to \mathtt{Y}$	proof $f\colon \mathtt{X} \to \mathtt{Y}$
monoidal product $\mathtt{X} \otimes \mathtt{Y}$	product data $\mathtt{X} \otimes \mathtt{Y}$	disjoint union $\mathtt{X} \otimes \mathtt{Y}$	joint systems $\mathtt{X} \otimes \mathtt{Y}$	conjunction $\mathtt{X} \otimes \mathtt{Y}$
monoidal product $f \otimes g$	parallel relations $f \otimes g$	disjoint union $f \otimes g$	parallel process $f \otimes g$	parallel proofs $f \otimes g$

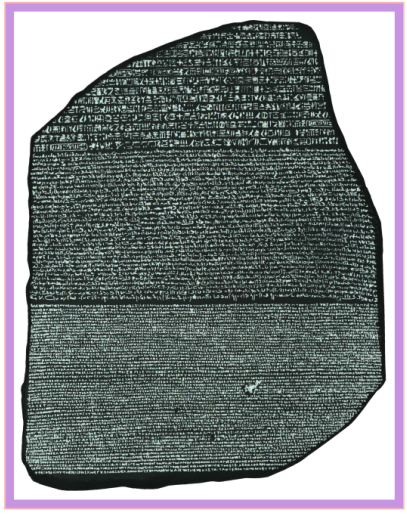
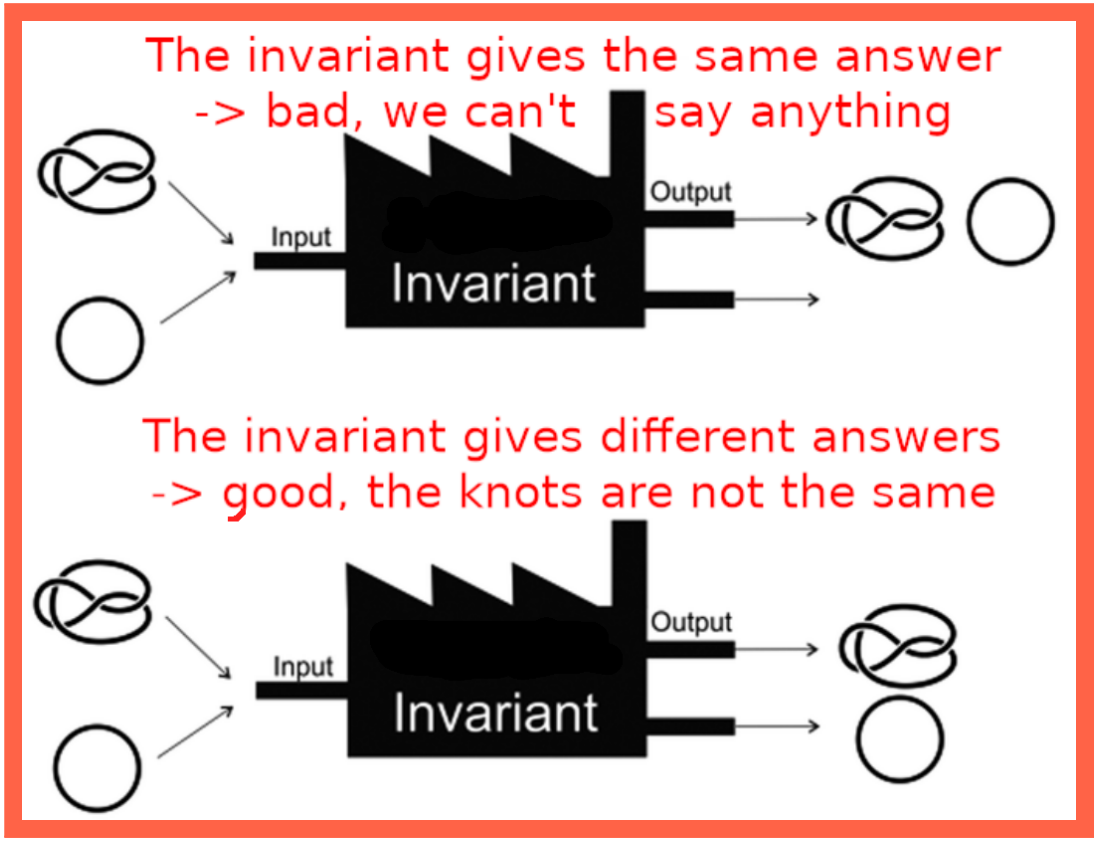


FIGURE 1. The Rosetta stone: the top and middle texts are in ancient Egyptian using hieroglyphic and Demotic scripts, respectively, while the bottom is in ancient Greek. The decree has only minor differences among the three versions, so the Rosetta stone became key to deciphering Egyptian hieroglyphs.

Picture from https://commons.wikimedia.org/wiki/File:Rosetta_Stone_BW.jpeg

- Above The Rosetta stone
- QT is a mixture between all of these
- Goal Use a categorical approach to explain other instances of the world

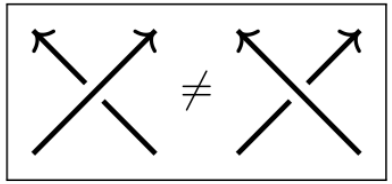
Key players: the quantum invariants



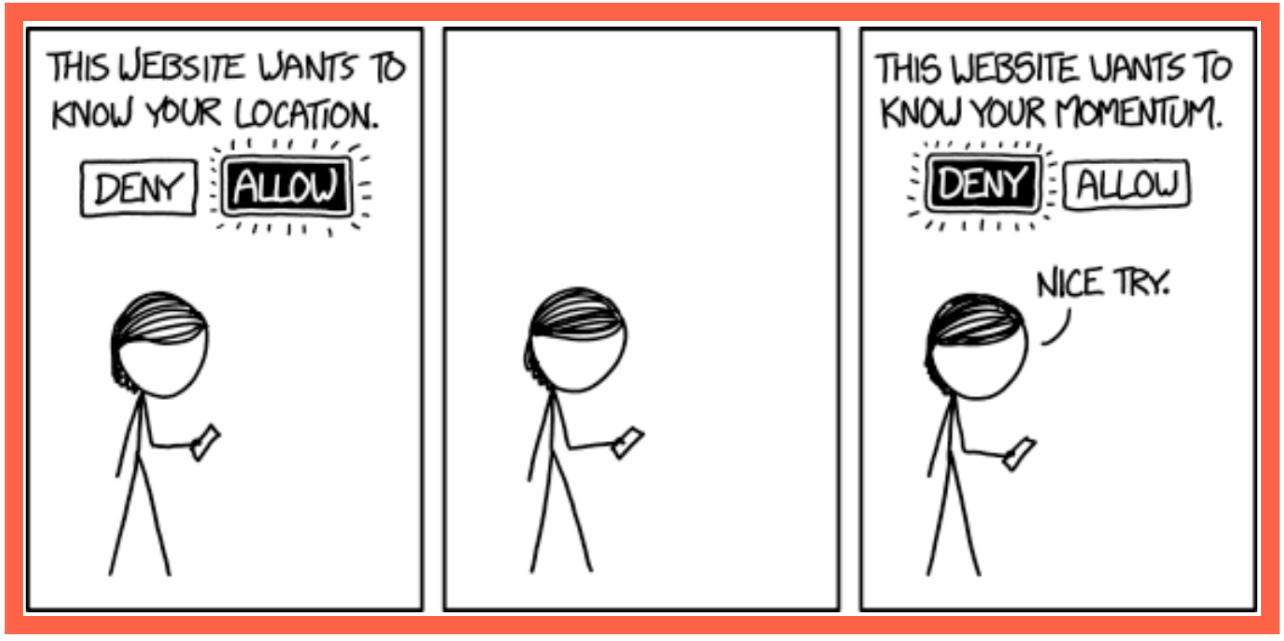
- Problem Low dimensional topology (knots, Poincaré conjecture, 2d, 3d, 4d) is very different from other parts of topology
- Topologists from ∼1900-1980 studied knots from the point of view of invariants from homology theory but these are not very strong
- Quantum invariants QT offered a new approach to study low dimensional topology

In QT: Quantum = noncommutative

Algebra: non-commutative structures
Topology: knots and links

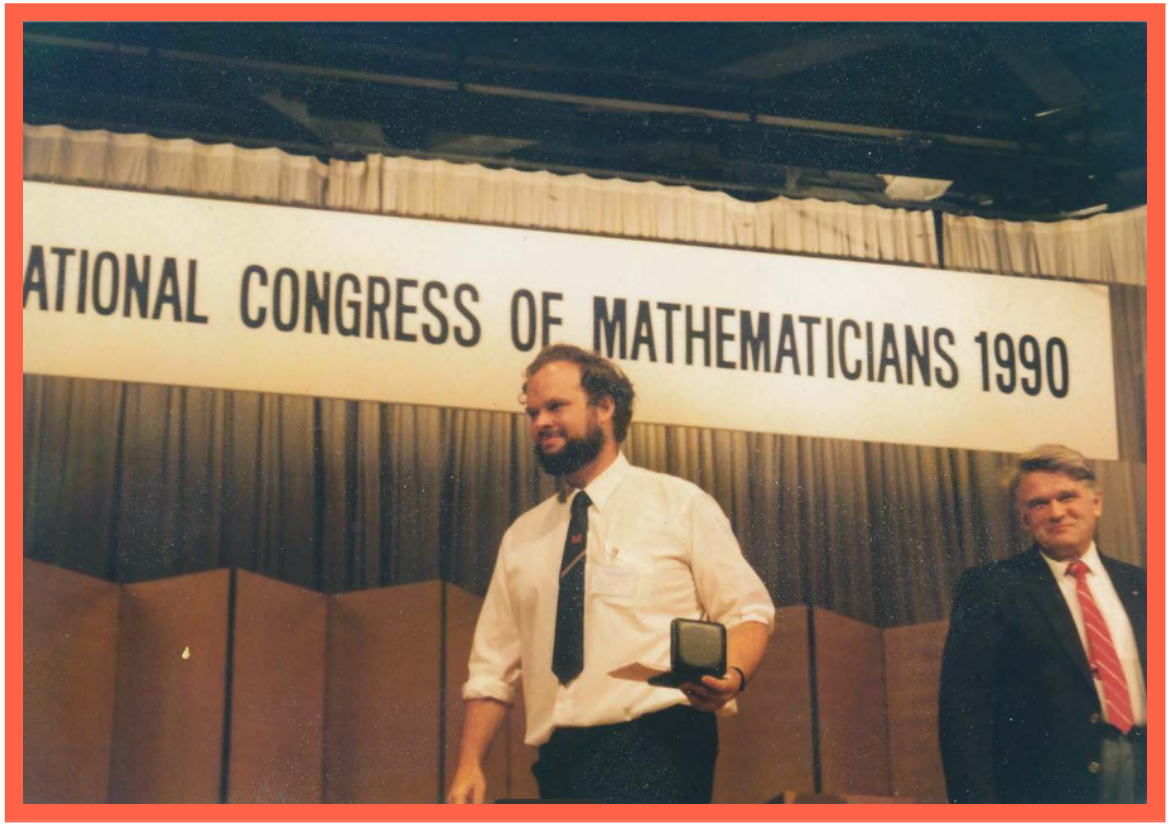


Physics: particles in $\mathbb{R}^3$
Logic: ribbon logic



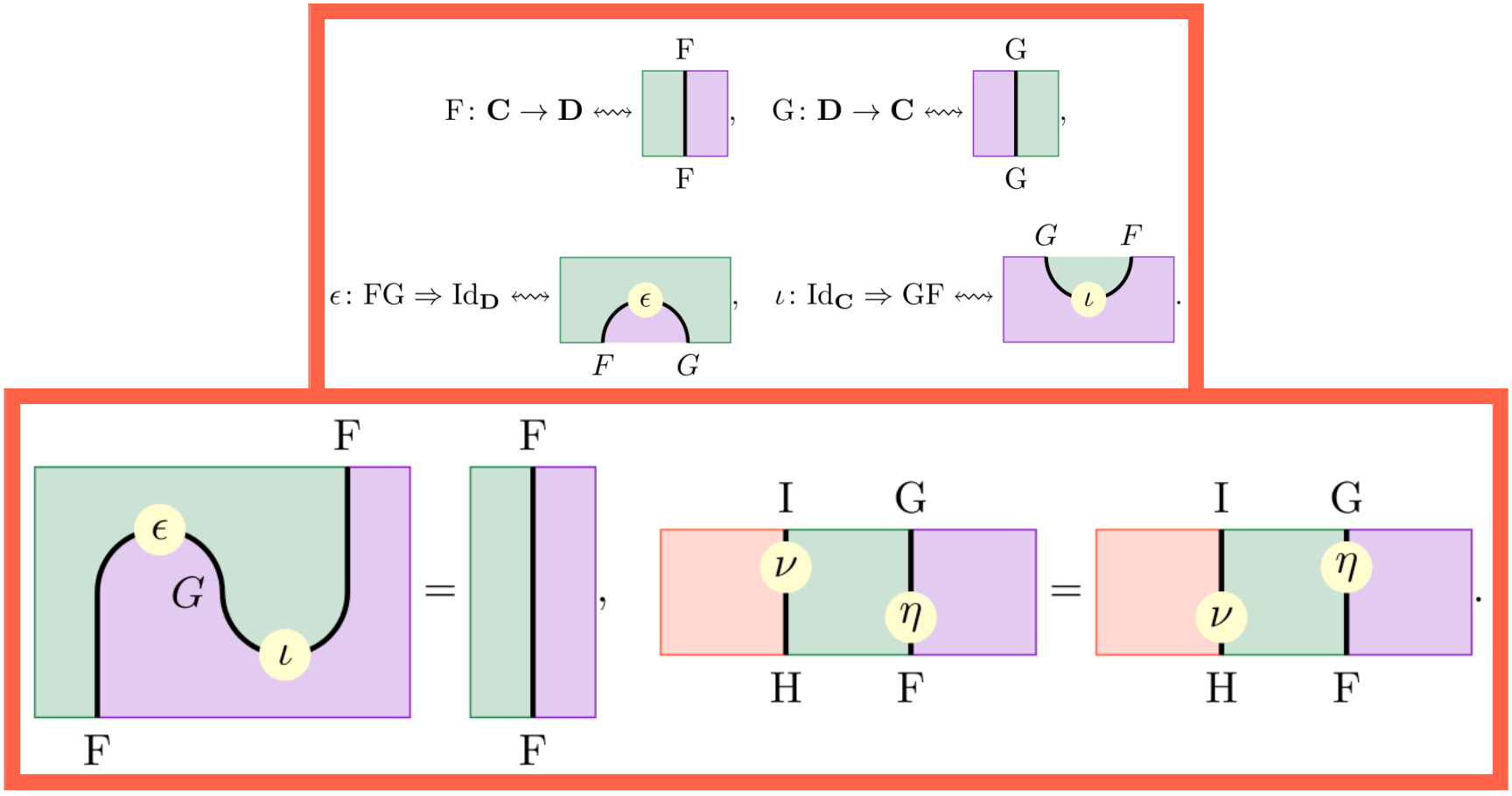
- Above (second) Heisenberg's uncertainty principle; crucial in quantum mechanics
- Essentially it says that measuring position-momentum does not commute
- QT is the study of noncommuting structures

What it can do for you



- Kyoto 1990 Jones receives the fields medal (with Faddeev in the background)
- Quote "Jones discovered an astonishing relationship between von Neumann algebras and geometric topology. As a result, they found a new polynomial invariant for knots and links in 3-space."
- Jones' revolution There are ∞ many (non-homological) invariants of knots etc.

How we will do it



- Approach : draw diagrams!
- Category theory = diagrammatics (at least for me ☺)
- Above Adjunction and interchange law diagrammatically

Thank you for your attention!

I hope that was of some help.