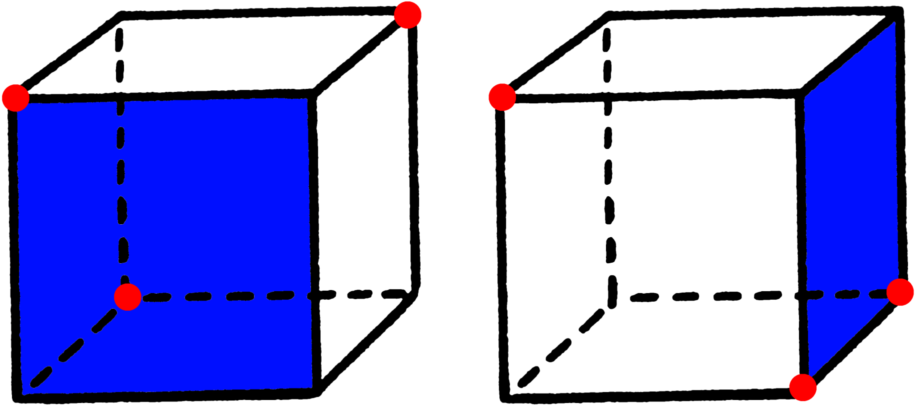
What is...Bondy's theorem?

Or: Forgetting without loss

From cubes to squares



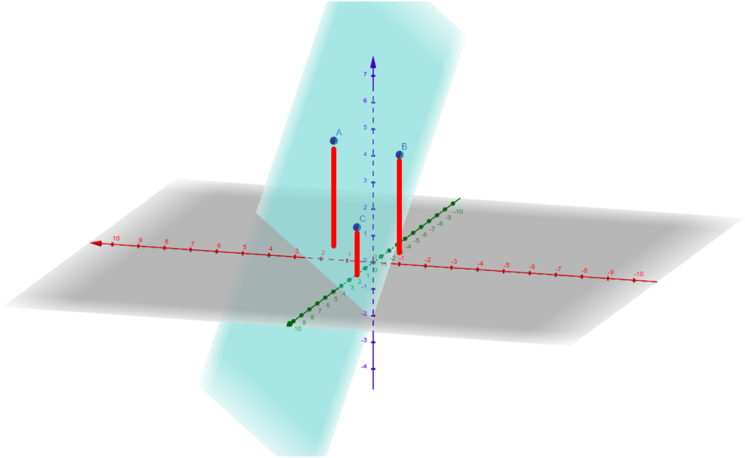
- Projecting the three points to the indicated planes keeps them unequal
- One checks that this is can be achieved for any three points
- Question Is something going on?

From 4x4 to 4x3 matrices

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

- Removing the indicated column/row keeps the rows/columns unequal
- The same works for any 4x4 matrix with 0-1 entries
- Question Is something going on?

Three points in $\mathbb{R}^3$



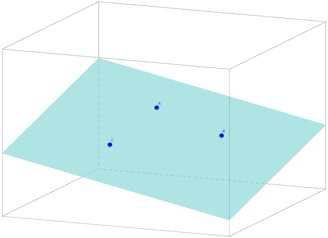
- Projecting the three points to the xy plane keeps them distinct
- Changing the projection plane, the same works for any three points
- Question Is something going on?

Enter, the theorem

Let S be a set with n elements and suppose that n distinct subsets of S are chosen. Then there is a restriction to $n-1$ elements of S under which these subsets remain distinct

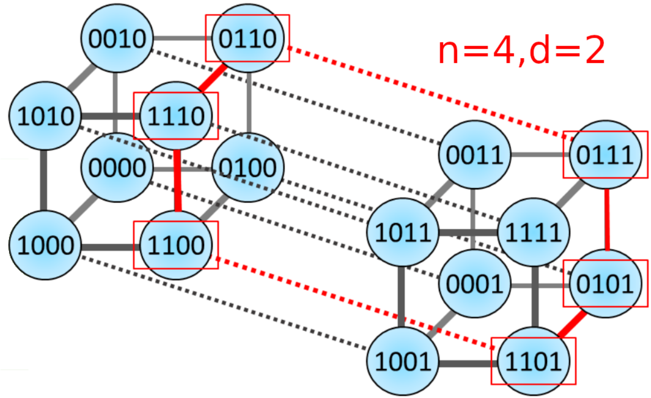
How could that be true?

- Think of n distinct vectors in $\mathbb{R}^n$
- Forgetting the ith coordinate identifies two $\Rightarrow$ the line between them is parallel to e_i
- No projection distinguishes them $\Rightarrow$ there are n pairwise orthogonal lines connecting them
- n points lie inside an $(n-1)$ dim subspace, *e.g.* three points in $\mathbb{R}^3$:



- Hmm, $n-1$ and n doesn't want to go along

Generalizing Bondy's theorem



- A version of Bondy's theorem Given a collection of distinct vertices on the n cube, what is the largest d such that some projection on $n-d$ dimensions results in a d cube?
- Above $d=2$
- This VC dimension d is an important concept in machine learning

Thank you for your attention!

I hope that was of some help.