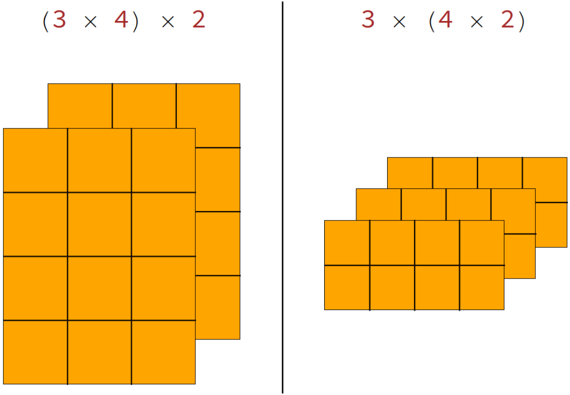
# What is...a coherence theorem?

Or: Don't underestimate associativity

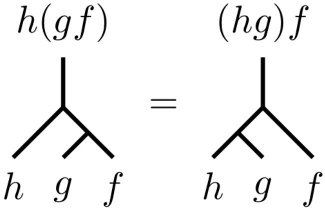
## Wait, this is not trivial!



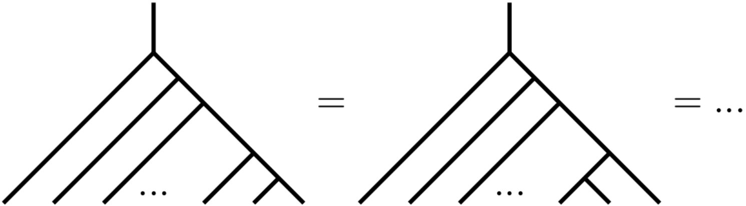

- **Associativity** $h \cdot (g \cdot f) = (h \cdot g) \cdot f$
- **Problem** This is not trivial, *e.g.* $(4/2)/2 = 1 \neq 4 = 4/(2/2)$
- **Even worse** Why should this imply *e.g.* $(i \cdot (h \cdot g)) \cdot f = (i \cdot h) \cdot (g \cdot f)$?

# A “wrong” and a “correct” definition

“Wrong” :

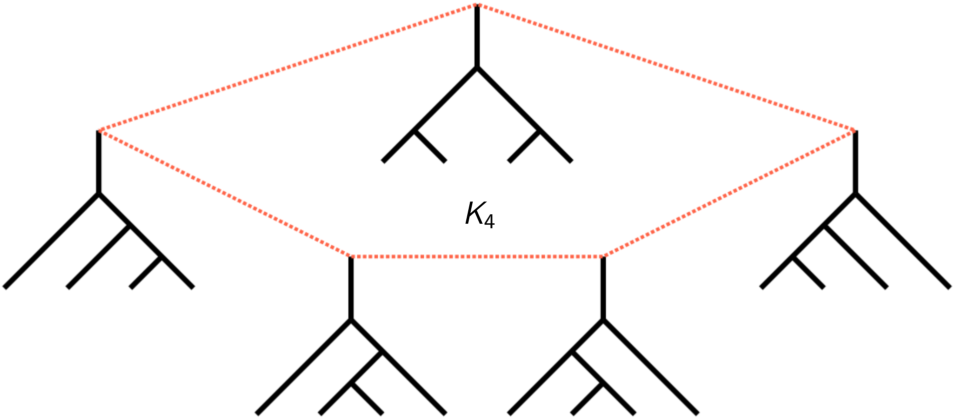



“Correct” :




- (A) $h \cdot (g \cdot f) = (h \cdot g) \cdot f$
- (B) Same result regardless of how valid pairs of parentheses are inserted
- “Philosophically correct” Use (B) as the definition and show that (A) $\Leftrightarrow$ (B)

## A proof that (A) ⇔ (B)




- **Vertices of $K_n$** All possible parenthesis of $n$ symbols
- **Edges of $K_n$** If there is a basic move connecting them
- **To show** $K_n$ is connected

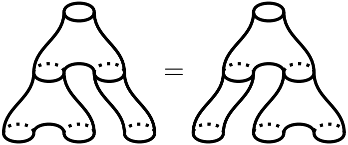
## Enter, the theorem

A coherence theorem is a theorem of the form

Finite collection of conditions $\Leftrightarrow$ All conditions

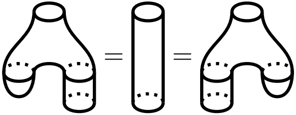
**Examples**

- Associativity $h \cdot (g \cdot f) = (h \cdot g) \cdot f \Leftrightarrow$ All bracketings



$\Leftrightarrow$ General pants move

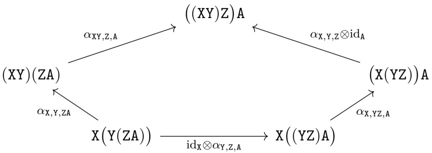
- Times 1 cancels $1 \cdot f = f = f \cdot 1 \Leftrightarrow$ All 1 cancels

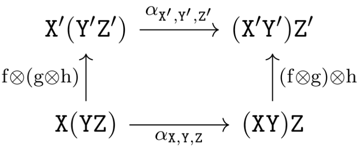


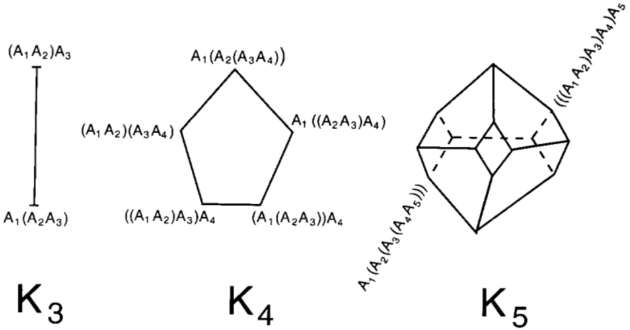
$\Leftrightarrow$ General absorption

- Mac Lane's famed coherence theorem for monoidal categories

# Coherence theorem for monoidal categories – a higher dimensional version




- Involves three pentagon, square and triangle (for the unit)
- Proof Show that $K_n$ has $\pi_1(K_n)$ trivial (for asso it was $\pi_0(K_n)$ trivial)

**Thank you for your attention!**

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I hope that was of some help.