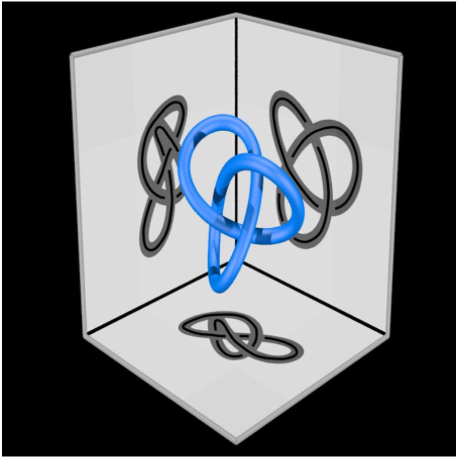
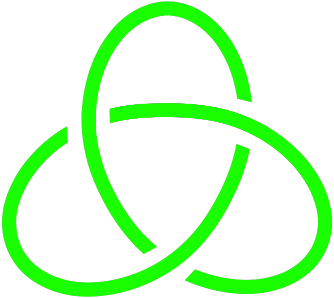
What is...the Jones polynomial?

Or: Resolving crossings

Knots, links and their projections

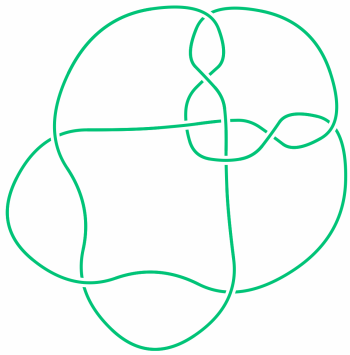


$\xrightarrow{\textit{project}}$



Question Do two projections represent the same knot?

The Perko pair



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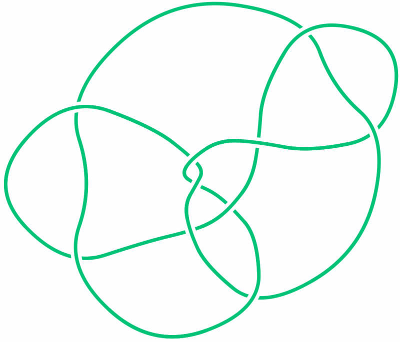
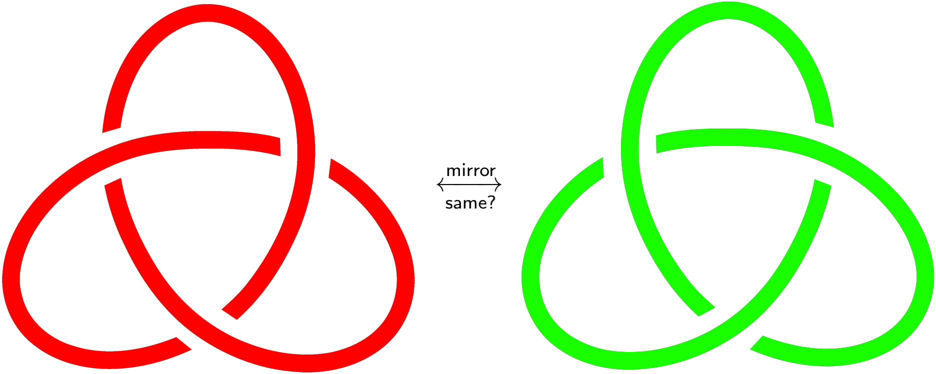


These are the same knot, but this took ages to be worked out!

The proof? Build them out of rope!

Problem Deciding whether two knot projections are the same knot is difficult

Are the mirror images the same?



Are these the same?

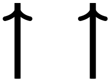
Problem Building them out of rope won't help...

Enter, the theorem

There is a polynomial invariant of oriented knots and link

$$V(\_)\colon \text{knots and link} \to \mathbb{Z}[q, q^{-1}]$$

satisfying the Skein relations

$q^{-1} \cdot V\Big($  $\Big) - q \cdot V\Big($  $\Big) = (q^{1/2} - q^{-1/2}) \cdot V\Big($  $\Big)$

(a) V is characterized by $V(\text{unknot}) = 1$ and the Skein relations

(b) $V(\text{alternating})$ is an alternating polynomial

(c) $V(L) =_{q \leftrightarrow q^{-1}} V(\text{mirror of } L)$

$V(\text{trefoil}) = -q^4 + q^3 + q$, so the trefoil is not equal to its mirror

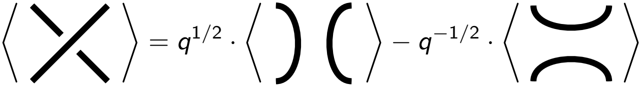
- From a worms perspective, V is powerful and easy at the same time
- From a birds perspective, V created a new field of mathematics, quantum topology, connecting various branches of modern mathematics and the sciences

On the back of an envelope – the Kauffman bracket

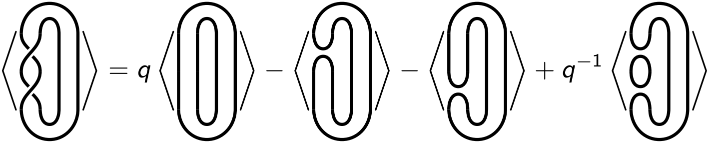
(a) $\langle \emptyset \rangle = 1$ Normalization

(b) $\langle \bigcirc \cup L \rangle = (q + q^{-1}) \cdot \langle L \rangle$ Pulling out circles

(c) Kauffman Skein



This gives the Jones polynomial up to normalization



$$= q(q + q^{-1})^2 - 2(q + q^{-1}) + q^{-1}(q + q^{-1})^2 = q^3 + q + q^{-1} + q^{-3}$$

Thank you for your attention!

I hope that was of some help.