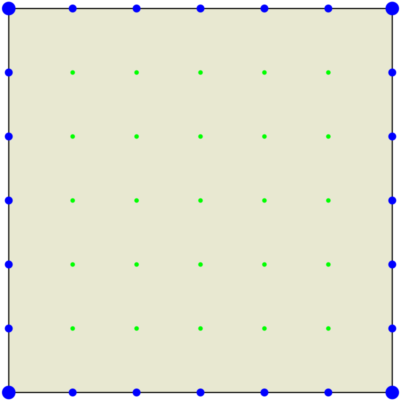
What is...Pick's theorem?

Or: Surprisingly simple

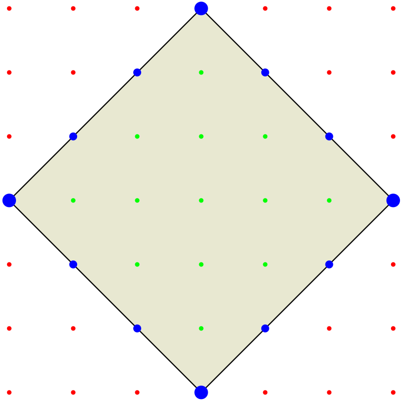
The area of a square



- A square P with corners on $\mathbb{Z}^2$
- Side length is 6 so $\text{area}(P) = 36$
- Note that $\text{area}(P) = \#B/2 + \#I - 1 = 24/2 + 25 - 1$, where B are the boundary points and I the interior points on $\mathbb{Z}^2$

Come on, this is obvious

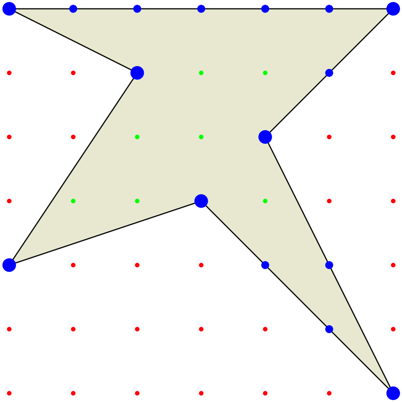
The area of another square



- A square P with corners on $\mathbb{Z}^2$
- Side length is $\sqrt{18}$ so $\text{area}(P) = 18$
- Note that $\text{area}(P) = \#B/2 + \#I - 1 = 12/2 + 13 - 1$, where B are the boundary points and I the interior points on $\mathbb{Z}^2$

Boring!

The area of a strange polygon



- A deformed 7-gon P with corners on $\mathbb{Z}^2$
- The computer tells me that $\text{area}(P) = 16$
- Note that $\text{area}(P) = \#B/2 + \#I - 1 = 16/2 + 7 - 1$, where B are the boundary points and I the interior points on $\mathbb{Z}^2$

Wait, what?

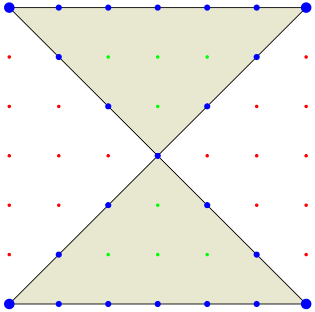
Enter, the theorem

P a simple polygon with integer coordinates for all of its vertices has area

$$\text{area}(P) = \#B/2 + \#I - 1$$

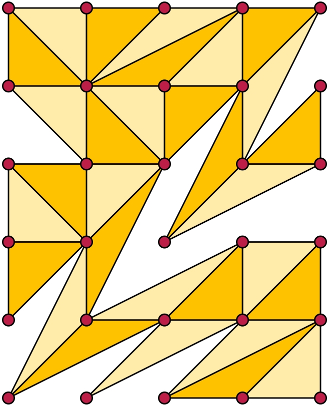
where B are the boundary points and I the interior points on $\mathbb{Z}^2$

- Simple means that P bounds a disk without self-intersection
- Pick's theorem is wrong if we drop the conditions, *e.g.*



has area 18, but $\#B/2 + \#I - 1 = 37/2$

Many proofs



- Pick's theorem has many proofs – as soon as you know what to prove it is not so hard
- A famous proof uses Euler's polyhedron formula $V - E + F = 1$ and the (easy) fact that triangles on $\mathbb{Z}^2$ with $\#I = 0$ are always of area $1/2$

Thank you for your attention!

I hope that was of some help.