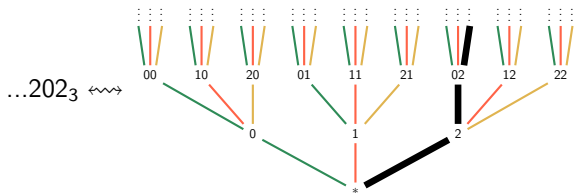
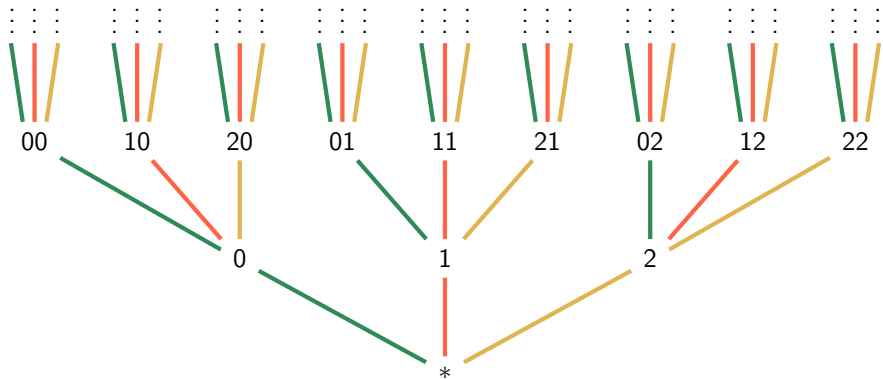


What are... p -adic integers?

Or: Climbing infinite trees

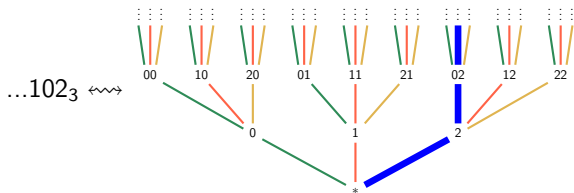
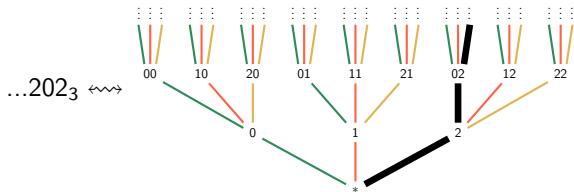
p -adic integers \mathbb{Z}_p are walks from the root to a leaf



Distance of walks in a tree

We have a tree metric $d(a, b) = 1/p^k$ with

k = distance of the first branching point to the root



$$d(\dots 202_3, \dots 102_3) = 1/3^2$$

Metric? Check! Addition 7-adically? Check!

Ansatz: 7-adic numbers are $(\dots a_2 a_1 a_0)_7$ for $a_k \in \{0, \dots, 6 = 7 - 1\}$

Addition of $\dots 251413_7$ and $\dots 121102_7$

Carry	...	1					
	...	2	5	1	4	1	3
+	...	1	2	1	1	0	2
<hr/>							
	...	4	7 = 0	2	5	1	5

What about $n \in \mathbb{Z}$? Apply subtraction from elementary arithmetic:

Carry	...	1	1	1	1	1	
	...	0	0	0	0	0	0
-	...	0	0	0	0	0	1
<hr/>							
	...	6	6	6	6	6	6

$-1 = \dots 666666_7$ in analogy to $1 = 0.999999\dots_{10}$

Enter, the theorem

p -adic integers \mathbb{Z}_p and numbers \mathbb{Q}_p exist via the equivalent definitions

(a) $\mathbb{Z}_p = \varprojlim \mathbb{Z}/p^n\mathbb{Z}$ and \mathbb{Q}_p is its field of fractions Algebra

(b) $\mathbb{Q}_p = \frac{(\text{Cauchy sequences in } \mathbb{Q} \text{ wrt } d)}{(\text{Nil sequences in } \mathbb{Q} \text{ wrt } d)}$ and \mathbb{Z}_p is its ring of integers Analysis

$\mathbb{R} = \frac{(\text{Cauchy sequences in } \mathbb{Q} \text{ wrt } d)}{(\text{Nil sequences in } \mathbb{Q} \text{ wrt } d)}$ for the standard metric Analogy

Theorem (local-global). Let $f \in \mathbb{Q}[X_1, \dots, X_n]$ be nice

(a) If $f = 0$ holds in \mathbb{Q} , then it holds in \mathbb{R} and \mathbb{Q}_p for all p global \rightarrow local

(b) If $f = 0$ holds in \mathbb{R} and \mathbb{Q}_p for all p , then it holds in \mathbb{Q} local \rightarrow global

Newton and $\sqrt{2}$ in p -adics

Solution by Newton's method of $x^2 - 2 = 0$

Suggestions: $(p, x_0) = (7, 3), (7, 4), (17, 6), (17, 11), (23, 5), (23, 18), (31, 8), (31, 23)$

n	x_n	x_n as fraction	x_n as p -adic	check: x_n^2 as p -adic
0	3	3	3_7	12_7
1	1.833- 33	$\frac{11}{6}$	$\dots 11111111111111113.0_7$	$\dots 32065432065432102.0_7$
2	1.462- 12	$\frac{193}{132}$	$\dots 33062113523306213.0_7$	$\dots 15156400343310002.0_7$
3	1.415	$\frac{72097}{50952}$	$\dots 01623525321216213.0_7$	$\dots 06010335100000002.0_7$
4	1.414- 21	$\frac{10390190017}{7346972688}$	$\dots 02011266421216213.0_7$	$\dots 10000000000000002.0_7$

$\sqrt{2} \approx \dots 216213.0_7$ 7-adically

Thank you for your attention!

I hope that was of some help.