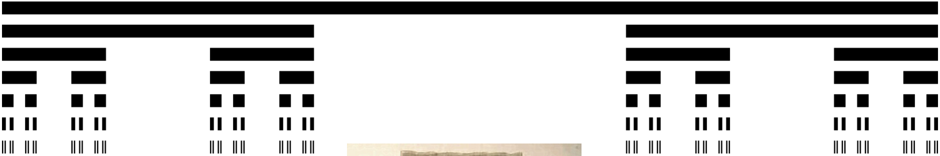
What is...the Cantor sequence?

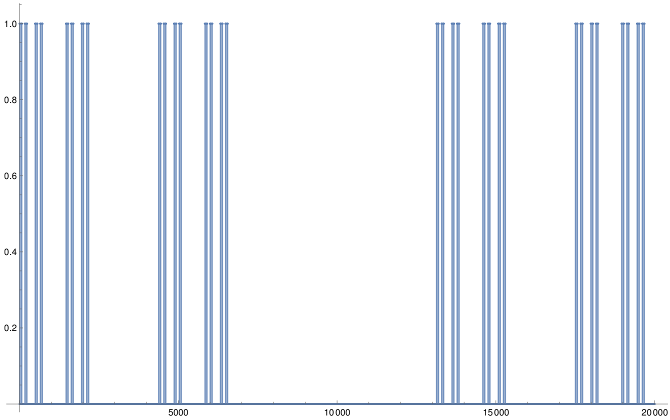
Or: 101000101...

Cantor's set



- ▶ Cantor's set = remove the middle third of a line segment and repeat
- ▶ This set is the prototype of a fractal; dimension is $\log_3 2 \approx 0.631$
- ▶ Task Make it discrete!

Cantor's sequence

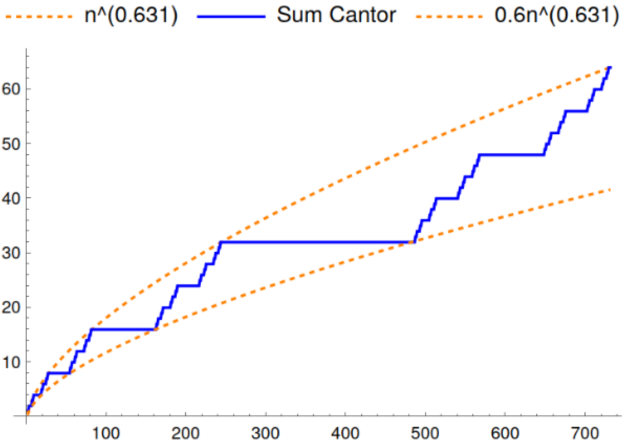


- Cantor's sequence ca_n:

$$\mathrm{ca}_n = \begin{cases} 1 & \text{if the ternary expansion of } n \text{ contains no 1,} \\ 0 & \text{otherwise.} \end{cases}$$

- This is like the Cantor set stretched over $\mathbb{N}$

Cantor's sum



- Whenever a sequence is going up and down take the sum $b_n = \sum_{k=1}^{n} ca_k$
- $b_n/n =$ average of the Cantor set sequence
- Task Understand b_n

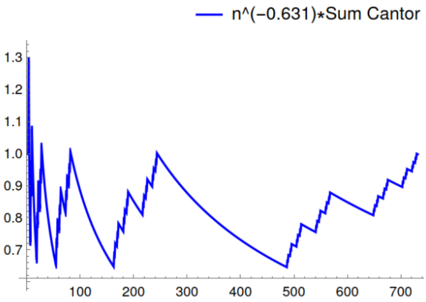
Enter, the theorem

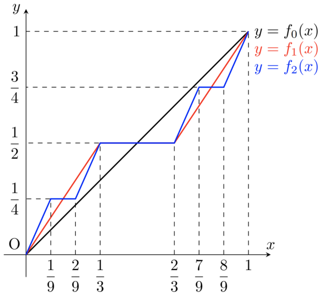
Asymptotically we have

$$b_n \sim h(n) \cdot n^{-\log_3 2}$$

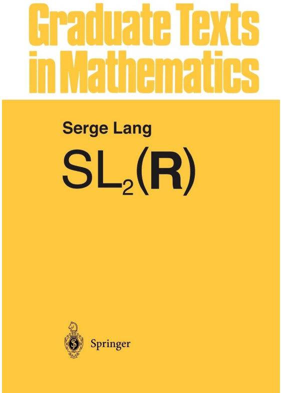
for a bounded function h

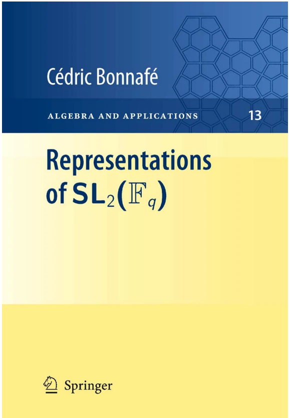
- The factor $\log_3 2$ is the dimension of the Cantor set
- h approaches ("is") devil's staircase (Cantor's function):





SL2, my friend





- SL2 $=$ 2-by-2 matrices with *det* $= 1$, say with entries in $\bar{\mathbb{F}}_3$
- ca_n $=$ sequence of weight space dimensions of a simple SL2 representation (well, of its distribution algebra)
- One can thus rediscover Cantor's XYZ from SL2 representation theory

Thank you for your attention!

I hope that was of some help.