What are...nearest neighbors?

Or: Close friends

Data clusters of words

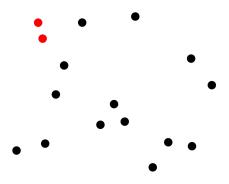


not this:



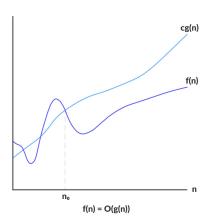
- ► How does a spell checker work?
- Simplified version
 - \triangleright We have a data base of words encoded as points in some \mathbb{R}^n
 - > The word we want to check is another point
 - □ Then we look for the closest neighbor point

The geometric reformulation



- ▶ Problem For *n* points in \mathbb{R}^d , find the pair that is closest
- ► The meaning of "closest" varies on the problem (e.g. this problem works in any metric space)
- Question How to find the closest pair efficiently?

A naive approach

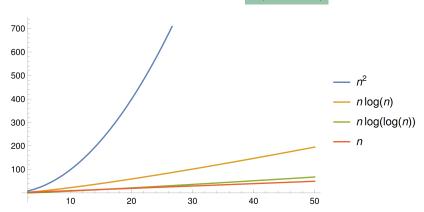


$$\blacktriangleright \text{ Recall } \sum_{k=1}^{n} k = n(n-1)/2 \approx n^2$$

- ► This naive algorithm measure all distances runs in $O(n^2)$
- ▶ Big O notation "Is bounded from above" see illustration

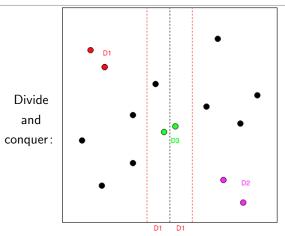
Enter, the theorem

Closest pair of points is in $O(n \log \log n)$



- $ightharpoonup n \log \log n$ is essentially linear
- ► Randomized algorithms can even solve this in linear time
- ► In the above we have the dimension *d* fixed

An $n \log n$ algorithm



- ▶ Divide Cut the problem in half and solve each half
- ▶ Safe time by only focusing on points within a region around the cut
- ► Conquer Do this recursively

Thank you for your attention!

I hope that was of some help.