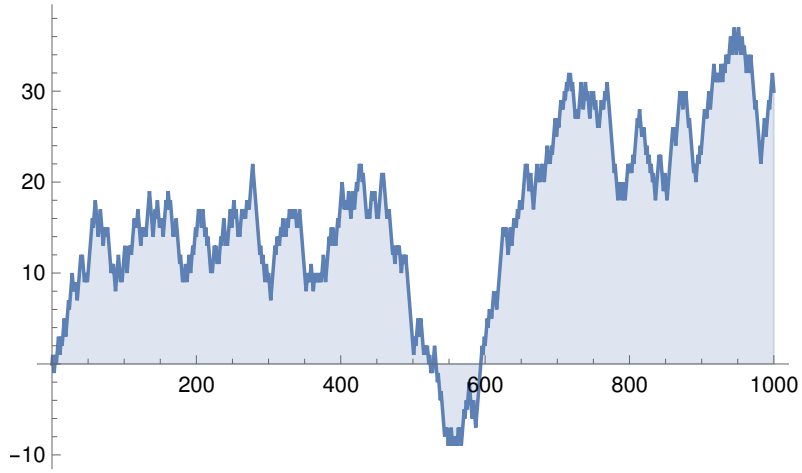
What is...the drunken bird constant?

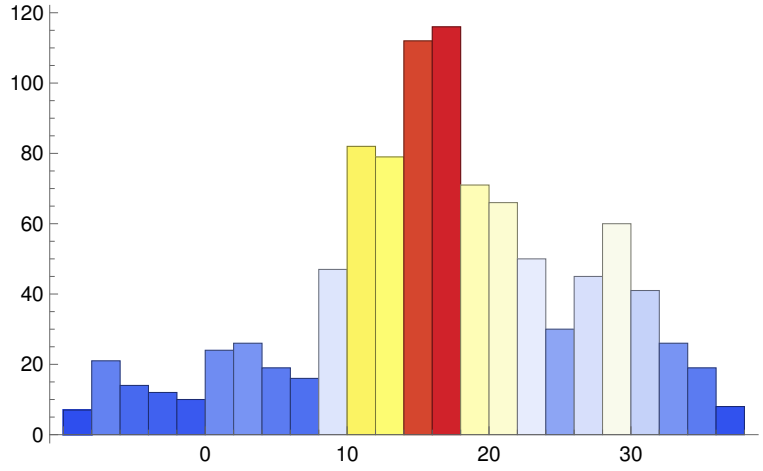
Or: Coming home, or not...?

1d random walk

A random walk:



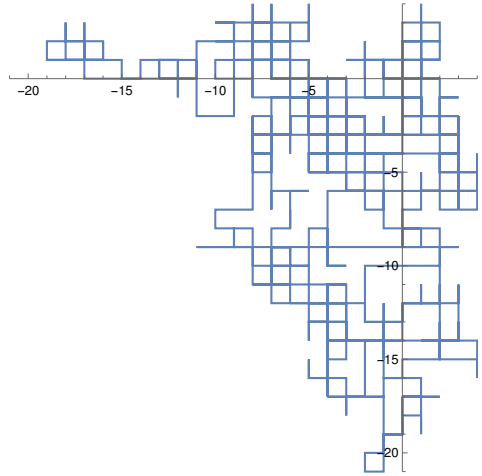
The distribution of how far one is from home :



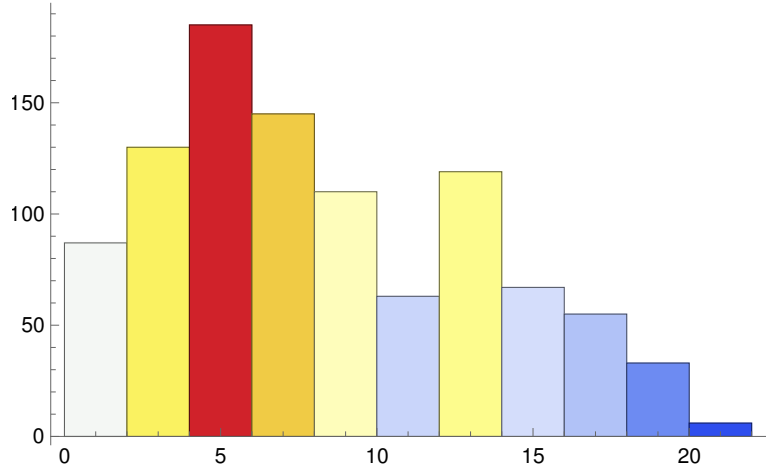
- 1d random walk = take a step left / right with probability $1/2$
- Question What is the probability p_{home} of return to the origin (=home)?
- Plotting this convinces one quickly that $p_{home} = 1$

2d random walk

A random walk:



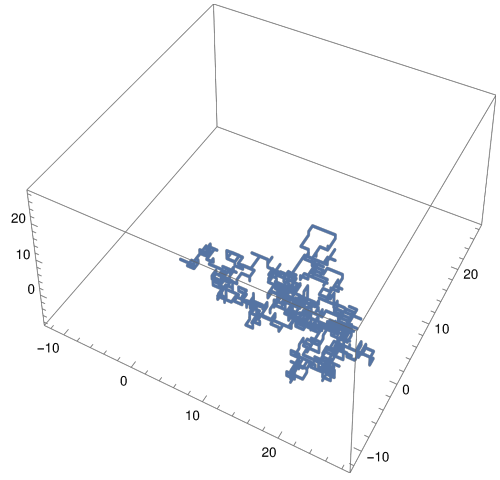
The distribution of how far one is from home :



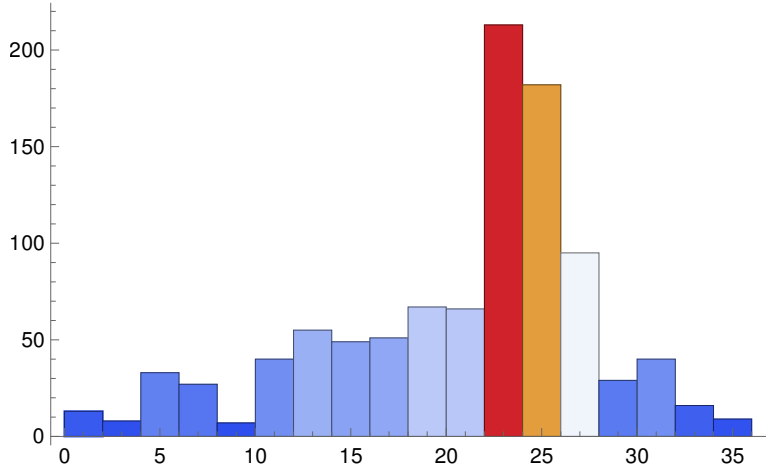
- 2d random walk = take a step left / right / up / down with probability 1/4
- Question What is the probability p_{home} of return to the origin (=home)?
- Plotting this gives a quite ambiguous result

3d random walk

A random walk:



The distribution of how far one is from home :



- 3d random walk = take a step left / right / up / down / in / out with probability 1/6
- Question What is the probability p_{home} of return to the origin (=home)?
- Plotting this convinces one quickly that $p_{home} < 1$

Enter, the theorem

We get the following p_{home} for dim $d > 2$:

$$p_{home} = 1 - 1/u(d)$$

with

$$u(d) = \frac{d}{(2\pi)^d} \int_{-\pi}^{\pi} \dots \int_{-\pi}^{\pi} \frac{1}{d - \cos x_1 - \dots - \cos x_d} dx_1 \dots dx_d$$

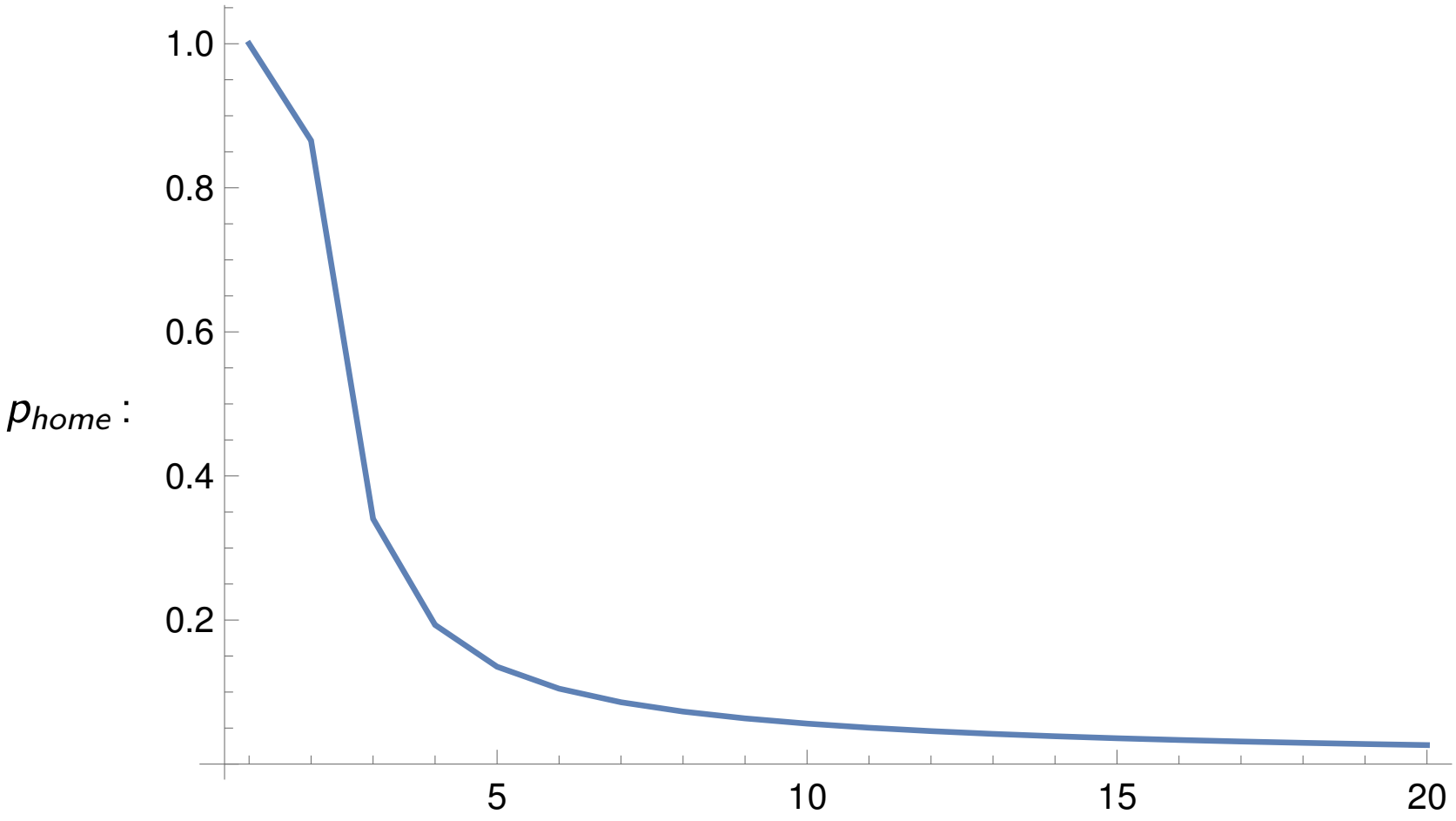
- For $d = 3$ the formula is

$$u(3) = \frac{3}{(2\pi)^3} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{1}{3 - \cos x - \cos y - \cos z} dxdydz$$

- Coming home? $d = 1, 2$ are special, and

$$p_{home} = 1 \text{ for } d = 1, 2$$

Coming home?



- Above dim on the x-axis, p_{home} on the y-axis
- The formula on the previous slide does not work for $d = 1, 2$
- But it kind of works for $d = 1$

Thank you for your attention!

I hope that was of some help.