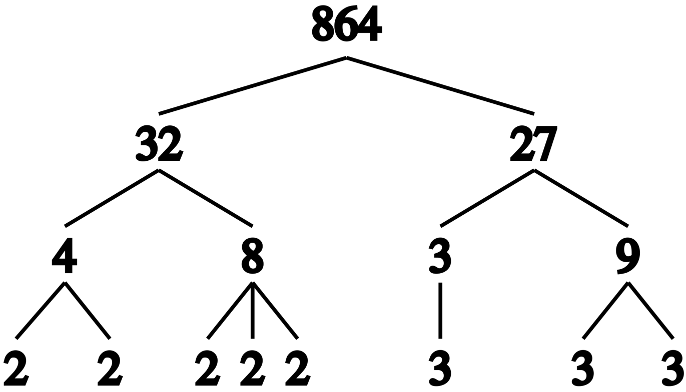
What is...Golomb–Dickman's constant?

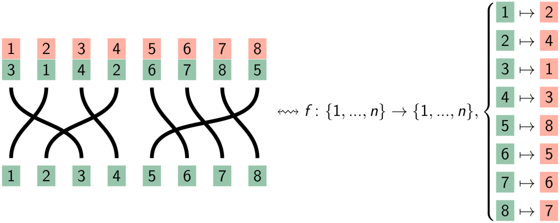
Or: Cycles and primes

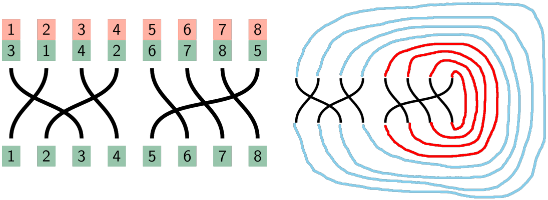
Factoring integers



- Prime factorization (PF) $n = p_1^{a_1} \ldots p_k^{a_k}$
- Precise properties of PF are often nasty, but statistical answers are often nice
- Example question What is the average size of the largest prime factor of n?

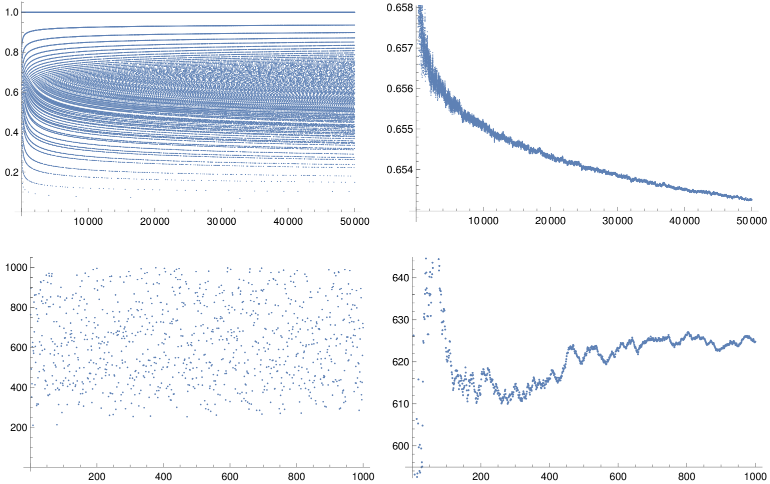
Factoring permutations





- **Cycle factorization (CF)** permutation of $\{1, \ldots, n\}$ = product of cycles
- Precise properties of CF are **often nasty**, but statistical answers are **often nice**
- **Example question** What is the average size of the largest cycle in S_n (= permutations of $\{1, \ldots, n\}$)?

Computer talk



- Top The average largest prime factor of n (in digit length)
- Bottom The average length of a longest cycle in S_{1000}

Enter, the theorem

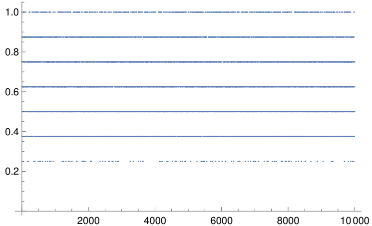
The counts come out as the same

$$\lambda = \lim_{n\to\infty} \frac{1}{n} \sum_{k=2}^{n} \frac{1}{\log k} \log P_1(k)$$

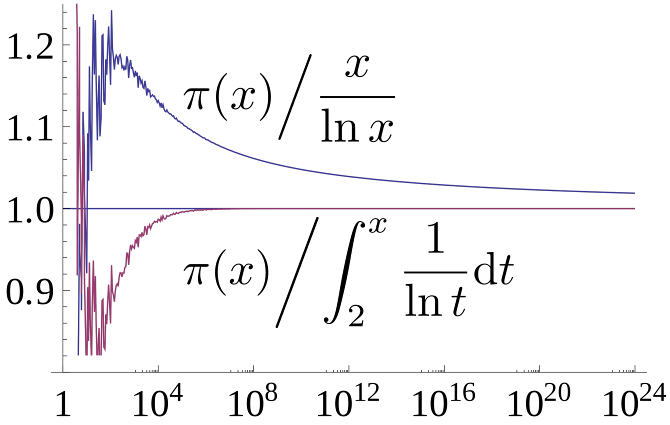
$$\lambda = \lim_{n\to\infty} \frac{1}{n} a_n$$

- $P_1(k)$ is the largest prime factor of k
- a_n = the average of the length of the longest cycle in each permutation in S_n
- $\lambda \approx 0.62$, so the average longest cycle makes up 62% of the maximal length, and ditto for PF (in digits)

Average size of largest prime factor for 10000 randomly selected numbers in $\{10000000, 20000000\}$:



The prime number theorem...kind of...



- We even have a precise formula:

$$\lambda = \int_0^1 e^{Li(t)} dt$$

- $Li(t) = \int_2^t 1/\ln(x) dx$ is the logarithmic integral

Thank you for your attention!

I hope that was of some help.