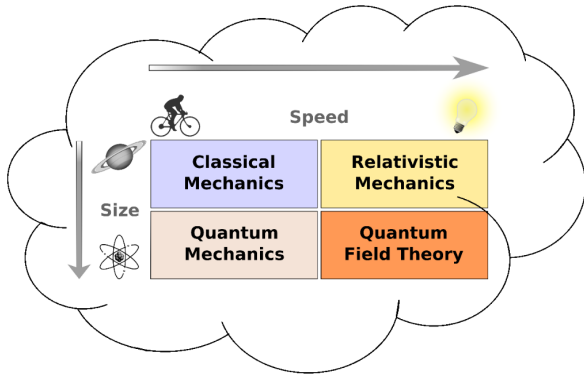**What is...the Yang–Mills mass gap?**

Or: One comes after zero

Quantum field theory (QFT)

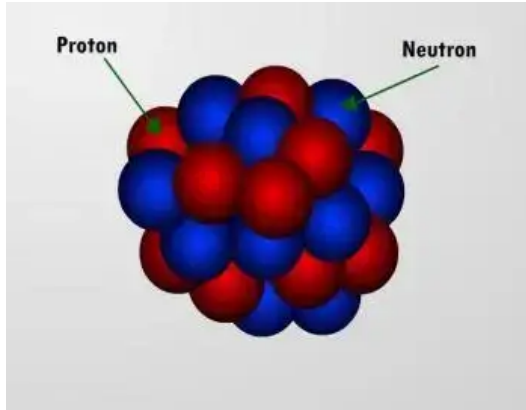
- ▶ QFT = the study of "the fast and the small"

- ▶ Example A combined theory of electromagnetic, weak and strong forces

- ▶ (Quantum) Yang–Mills theory (YMT) aims to describe the behavior of elementary particles under these forces

Gauge theories – abelian vs. nonabelian

(q)YMT is
a theory of :
the nucleus

- ▶ (q)YMT is a $\boxed{\text{gauge theory}}$ – a model using a Lie group G of symmetries (e.g. electroweak force is a $U(1) \times SU(2)$ (q)YMT; strong force a $SU(3)$ (q)YMT)

- ▶ What this generalizes: $\boxed{G \text{ abelian}}$ (e.g. ordinary electromagnetism)

- ▶ What one is up for: $\boxed{G \text{ nonabelian}}$ (e.g. standard model)

Mass gap


- Mass gap $=$ particles have positive mass, even though they travel at the speed of light (quantum phenomena)
- Example The strong force is strong but short-ranged due to a mass gap
- Many predictions of YMT are experimentally verified (real-world, computer) – but the mass gap is experimentally verified but not part of classical YMT

Millenniums price problem   Existence of nontrivial 4d qYMT with a mass gap

Example-based description :

(i) QCD (quantum chromodynamics) is the QFT of YMT describing the quantum theory of gluons and quarks (strong force); gauge group is $SU(3)$
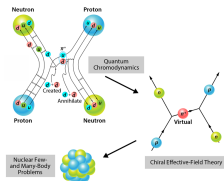

(ii) Theorem (experiment/computer) QFT has a mass gap (at not normal temp.)

(iii) Mass gap problem = demonstrate theoretically the existence of a mass gap for QCD or more general qYMT

▶ Mass gap mathematically = a certain operator has spectrum in $(\Delta > 0, \infty)$

▶ Classical YMT = a collection of differential equations is fairly well-understood

Difficulties of renormalization

Standard Model of Elementary Particles
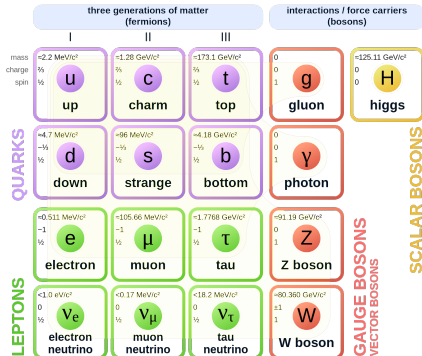
- ▶ At present there is `no satisfactory` mathematical definition of qYMT

- ▶ Establishing `certain axioms` is the "existence" part of the problem; the mass gap is an additional property; the mass gap is an `additional property`

- ▶ `Importance` Foundation of the standard model and cool math thingies

Thank you for your attention!

I hope that was of some help.