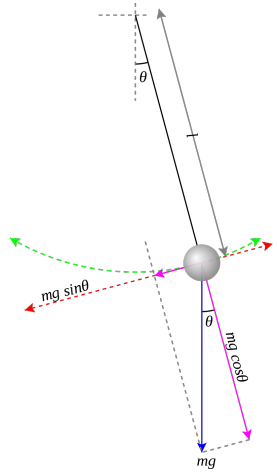
What are...the Navier–Stokes equations?

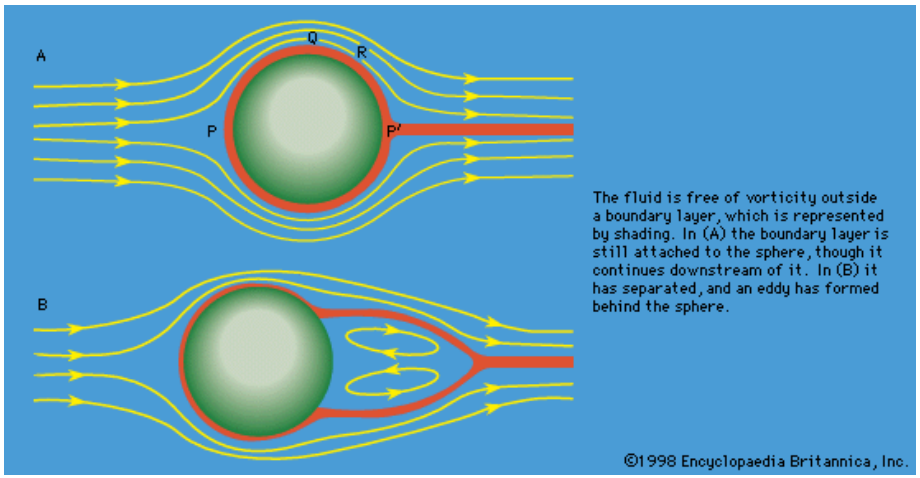
Or: Fluids are difficult

The system evolves

Pendulum–easy:

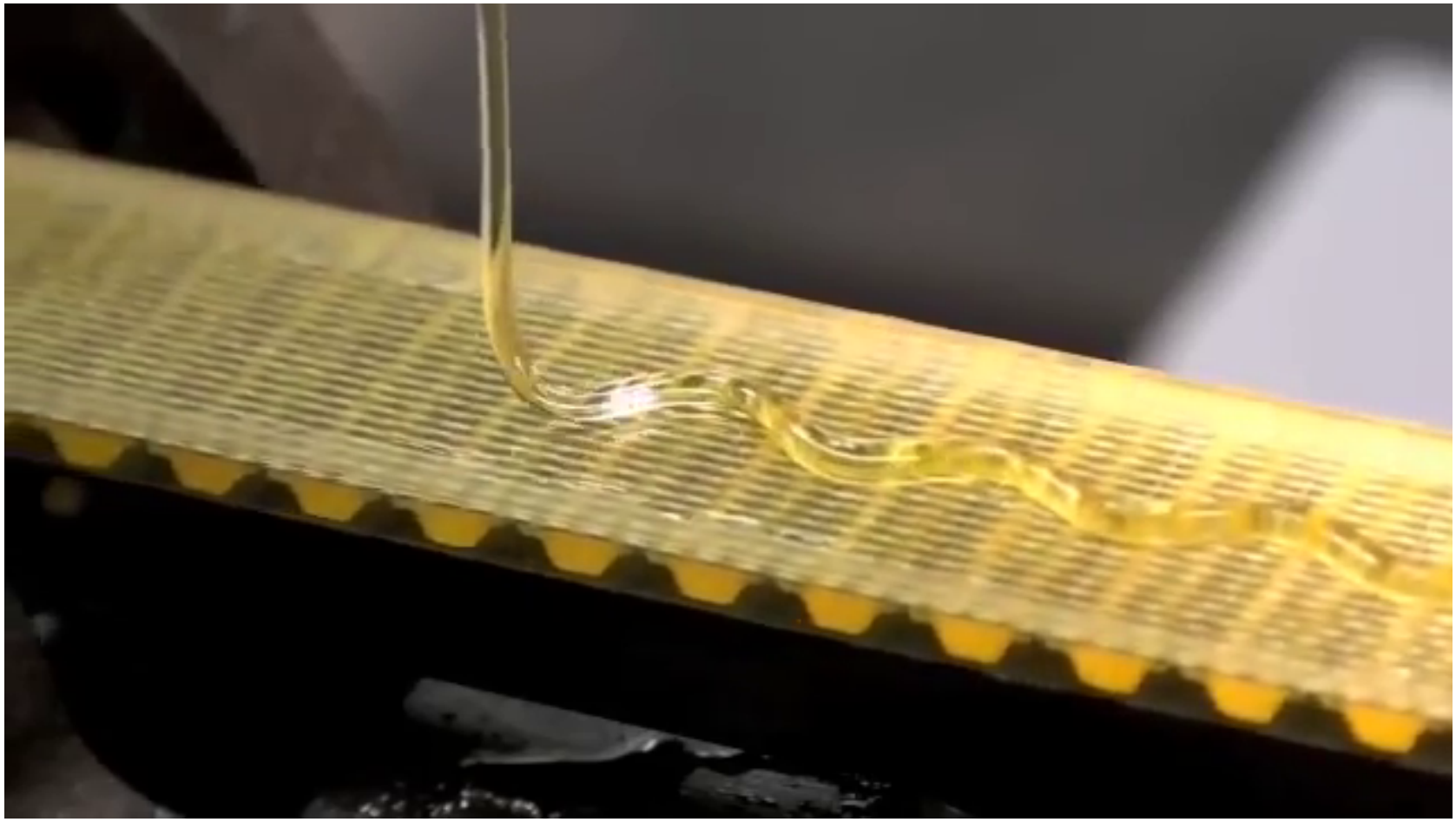


Fluid–difficult:



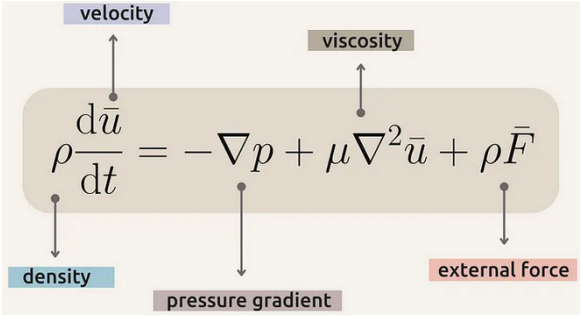
- Differential equations (DE) describe the changes of a system with passes
- Example Description of the pendulum or the behavior of fluids
- Roughly Easy/complicated systems are easy/impossible to solve as DE

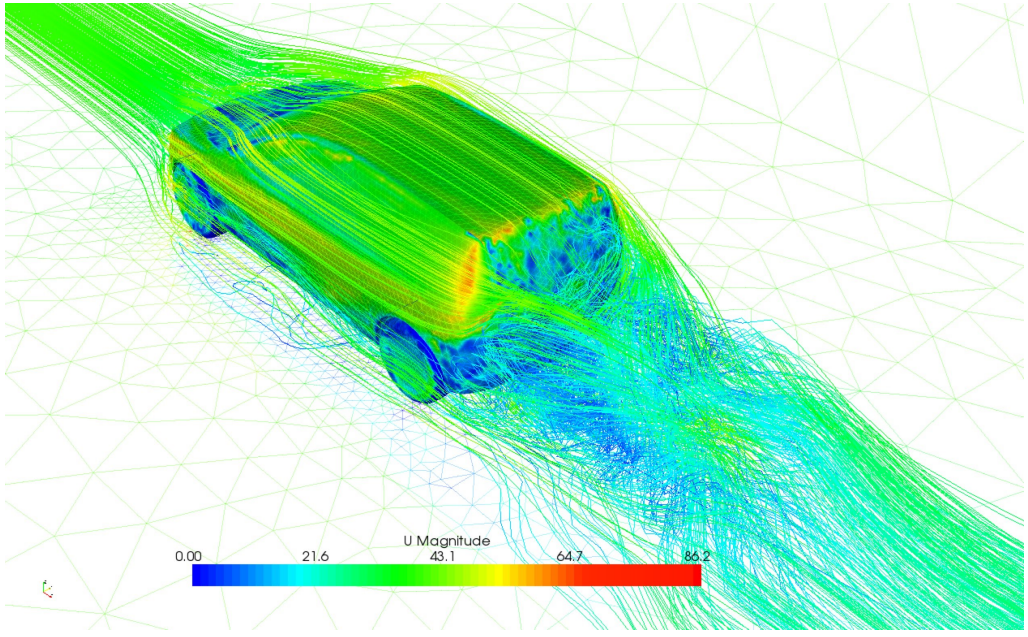
Sometimes its too difficult...



- Most real-world systems have probably no nice solution
- In other words, most DE are hard to impossible to solve
- Maybe Attack DE numerically or at least show existence of some nice answer

Fluids and friends





- Navier–Stokes equation (NSE) ⇜⇝ the motion of "viscous fluid substances"
- They are everywhere in engineering
- Question They are fairly easy to get, but what about solutions?

Enter, the theorem

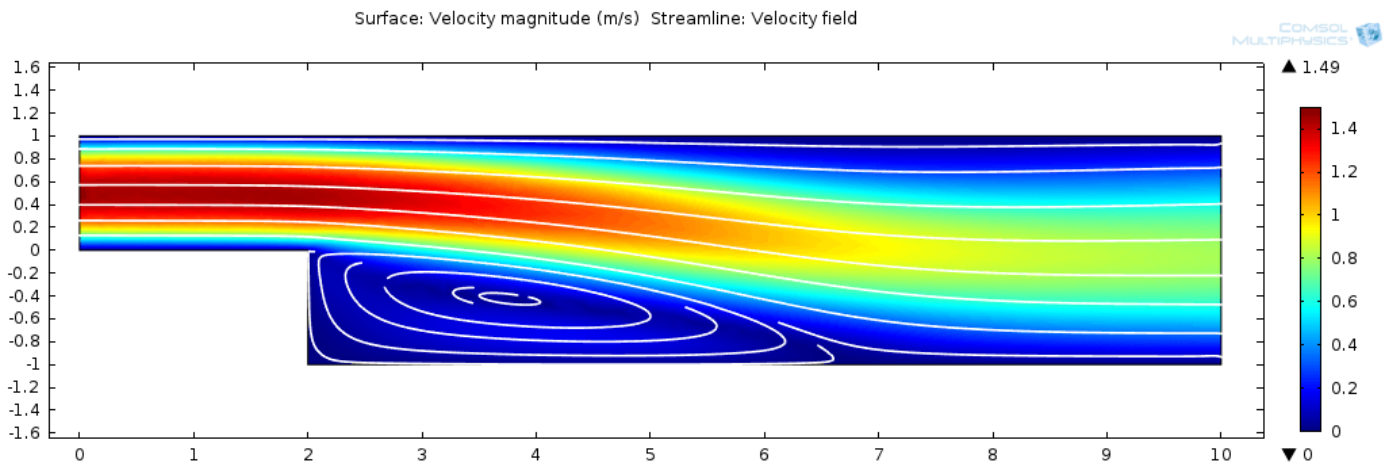
Regarding the NSE:

(i) Some exact solutions exist

(ii) Numerical solutions are well-known and quite good

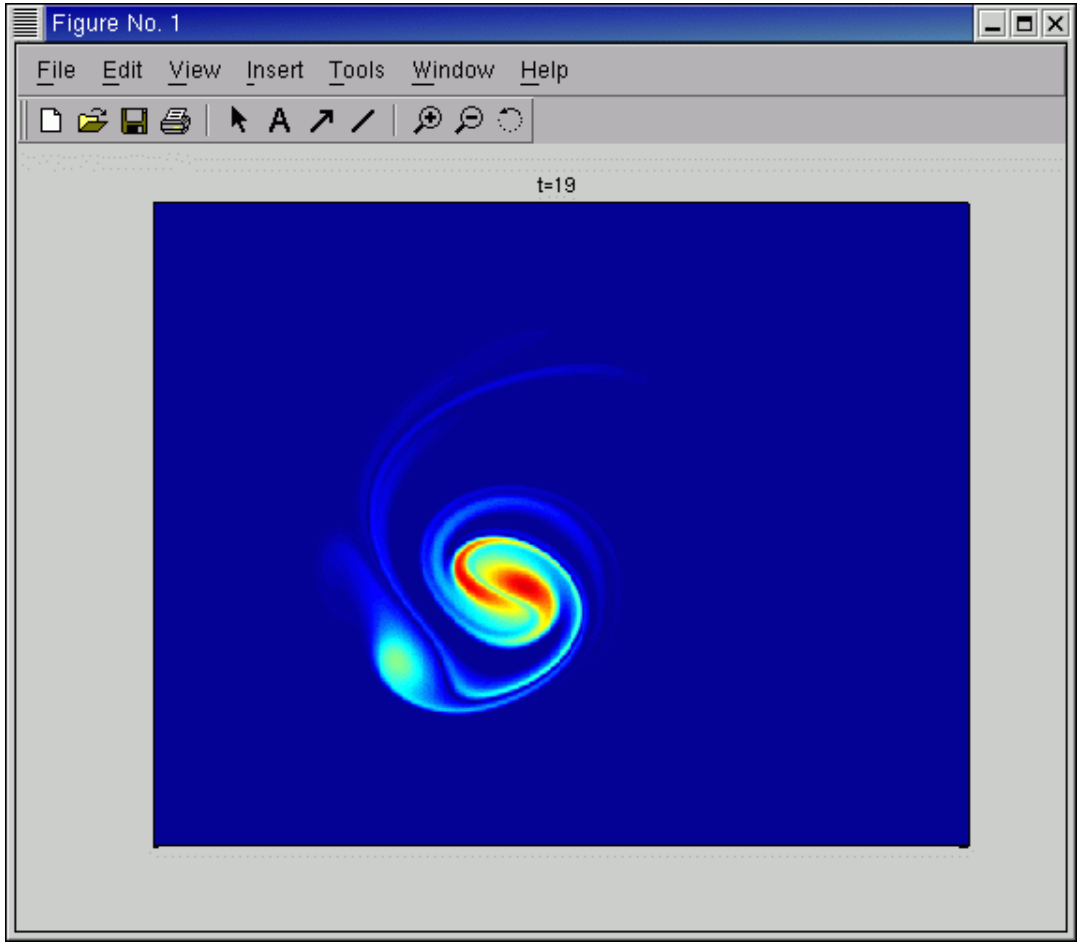
(iii) Weak solutions exist (satisfying the NSE in mean value, not pointwise)

(iv) 2d solutions exist (meaning smooth solutions)



- Millenniums price problem Do smooth solutions always exist in 3d?
- This is more a theoretical question, and not so much of practical importance

Real world implications? Well...



- Numerical solutions exists
- Many computer algebra systems have them built-in
- This is usually enough for practical purposes

Thank you for your attention!

I hope that was of some help.