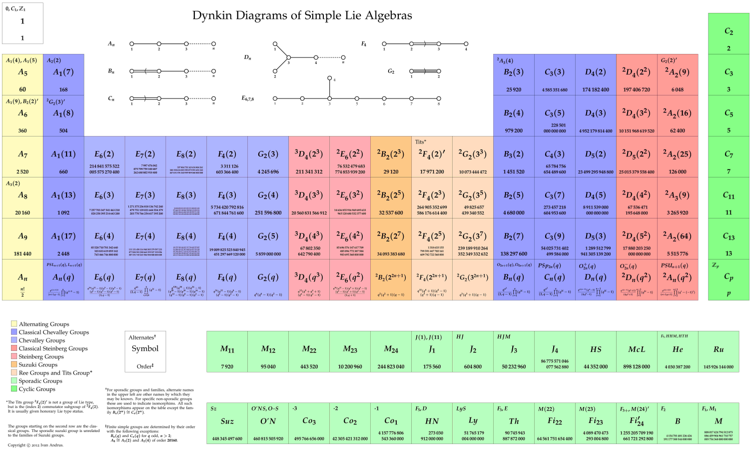
What are...sporadic groups?

Or: Weird, but interesting

The periodic table

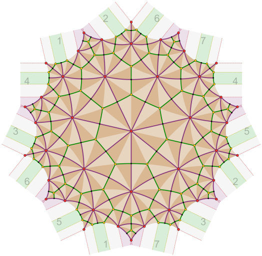


- Simple group = a group without substructure (normal subgroup) = an element
- Periodic table of group theory = classification of finite simple groups; one of the main math achievements of the 20th century
- Roughly Everything is a group Lie-type except some sporadic examples

The Mathieu groups

					$J(1), J(11)$	HJ	HJM				F_7, HHM, HTH	
M_{11}	M_{12}	M_{22}	M_{23}	M_{24}	J_1	J_2	J_3	J_4	HS	McL	He	Ru
7 920	95 040	443 520	10 200 960	244 823 040	175 560	604 800	50 232 960	86 775 571 046 077 562 880	44 352 000	898 128 000	4 030 387 200	145 926 144 000

Sz	$O'NS, O{-}S$	·3	·2	·1	F_5, D	LyS	F_3, E	$M(22)$	$M(23)$	$F_{3+}, M(24)'$	F_2	F_1, M_1
Suz	$O'N$	Co_3	Co_2	Co_1	HN	Ly	Th	Fi_{22}	Fi_{23}	Fi'_{24}	B	M
448 345 497 600	460 815 505 920	495 766 656 000	42 305 421 312 000	4 157 776 806 543 360 000	273 030 912 000 000	51 765 179 004 000 000	90 745 943 887 872 000	64 561 751 654 400	4 089 470 473 293 004 800	1 255 205 709 190 661 721 292 800	4 154 781 481 226 426 191 177 580 544 000 000	808 017 424 794 512 875 886 459 904 961 710 757 005 754 368 000 000 000

$M_{24} = Sym\Bigg($  $\Bigg)$ + extra symmetry

- The groups of Lie-type were well-known for a long time
- Essentially in parallel, Mathieu ~1861 discovered the first sporadic groups
- Construction Augmented symmetries of naturally appearing objects

Enter, Janko

					$J(1), J(11)$	HJ	HJM				F_7, HHM, HTH	
M_{11}	M_{12}	M_{22}	M_{23}	M_{24}	J_1	J_2	J_3	J_4	HS	McL	He	Ru
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Sz	$O'NS, O\text{–}S$	$\cdot 3$	$\cdot 2$	$\cdot 1$	F_5, D	LyS	F_3, E	$M(22)$	$M(23)$	$F_{3+}, M(24)'$	F_2	F_1, M_1
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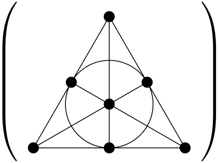
2. ^ The group theorist Bertram Huppert said of J_1: "There were a very few things that surprised me in my life... There were only the following two events that really surprised me: the discovery of the first Janko group and the fall of the Berlin Wall."

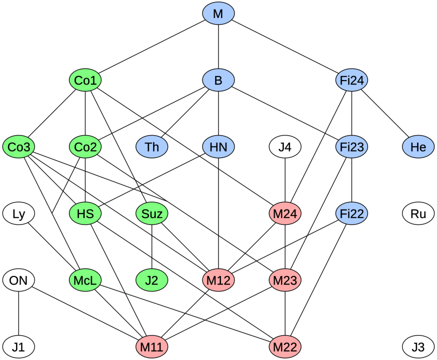
$J_2 = Sym$  + extra symmetry

- **Janko ~1965** constructed the first sporadic groups since Mathieu
- **Golden era** All other sporadic groups were discovered within 20 years
- **Construction** Augmented symmetries of naturally appearing objects

Enter, the theorem

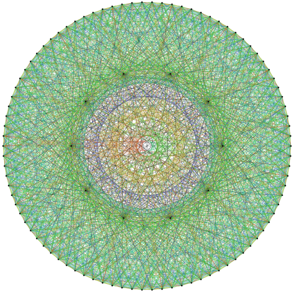
There are precisely 24 sporadic groups

- There is thus a largest sporadic group, the so-called monster
- Construction Augmented symmetries of naturally appearing objects
- Almost all sporadic groups are subgroups of the monster group M and $M =$ "symmetries of a certain monster Lie algebra"



Constructing J_2

$J_2 = Sym$ () $+$ extra symmetry

$J_2 = Sym$ ()

- J_2 is a index 2 subgroup in the automorphisms of a graph HJ on 100 vertices
- Construction Take the Fano plane with 7 points/lines, add 1 dummy + 21 flags (point-line pair) $\Rightarrow$ the Fano graph on 36 vertices
- Construction–continued Add another dummy and 63 vertices coming from 63 involutions of $U(3,3)$

Thank you for your attention!

I hope that was of some help.