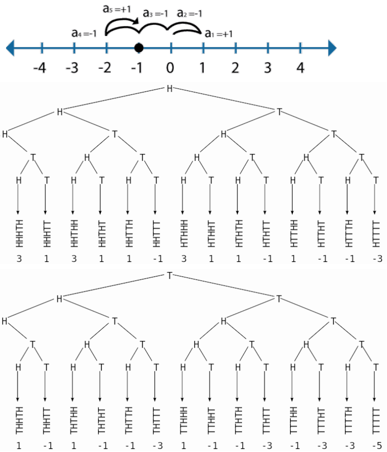
What is...recurrent versus transient?

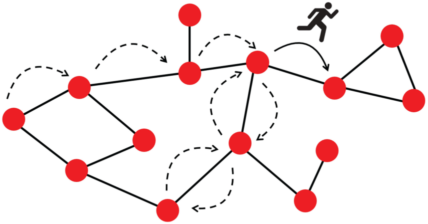
Or: Leaving or staying

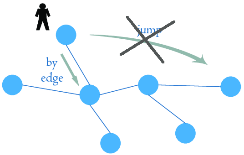
Coin flip walks



- Coin flip random walk = flip a coin and walk left for tails and right for heads; think of walking on the graph $\mathbb{Z}$
- Question How often do we visit the a vertex?
- Recurrent We will hit every point infinitely often with P(robability)=1

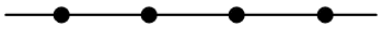
Random walks on general graphs



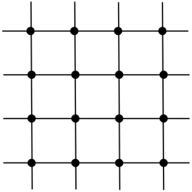


- We randomly walk on some (connected) graph = at each step choose the next step/edge randomly
- We only consider the case where every edge is equally likely to be chosen, and ask the same question as on the previous slide
- Example Every (random walk on a) finite graph is recurrent

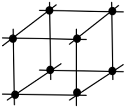
Transient



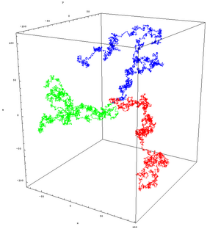
1-dimensional lattice



2-dimensional lattice



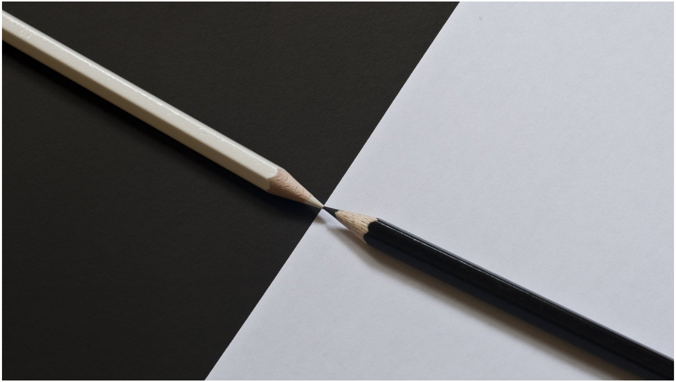
3-dimensional lattice



- **Polya's theorem** $\mathbb{Z}^d$ is recurrent/transient $\Leftrightarrow$ $d \leq 2/d > 2$
- A drunkard will find their way home, but a drunken bird may get lost forever
- **Transient** We will hit every point **finitely often with P(robability)=1**

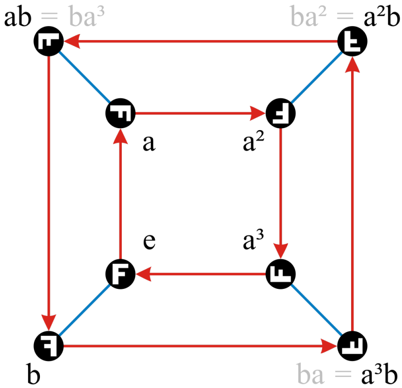
Enter, the theorem

Every graph is either recurrent or transient



- The are not *a priori* opposites: *a priori* there could be graphs with P=0.5
- This is an instance of a 0-1-theorem: a lot of properties hold with P=0 or P=1 but $0<P<1$ rarely appears

Recurrent/transient groups



- Call a group recurrent/transient if its Cayley graph is recurrent/transient
- Theorem A group is either recurrent or transient, and recurrent $\Leftrightarrow$
  - (i) Its finite
  - (ii) It virtually $\mathbb{Z}$ or $\mathbb{Z}^2$ (= it contains $\mathbb{Z}$ or $\mathbb{Z}^2$ with finite index)

Thank you for your attention!

I hope that was of some help.