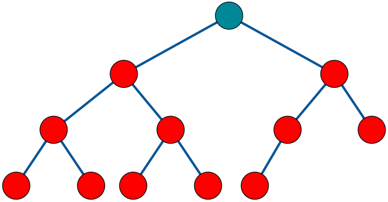
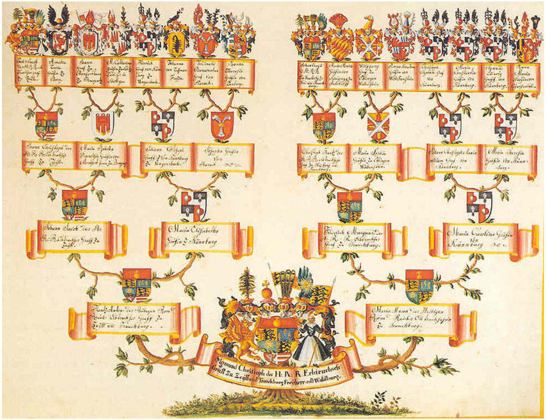
What are...tree rotations?

Or: A surprisingly small bound

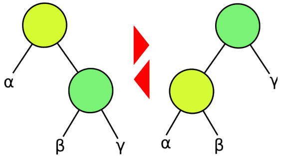
Binary trees

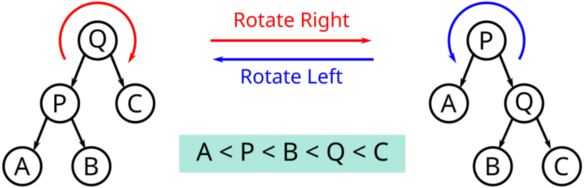




- Informally Binary tree = a form of a Ahnentafel (ancestor table)
- Binary tree = root/colored vertex joined to either zero or two subtrees, each of which is again a binary tree
- Often but not always these are drawn with the root vertex at the top

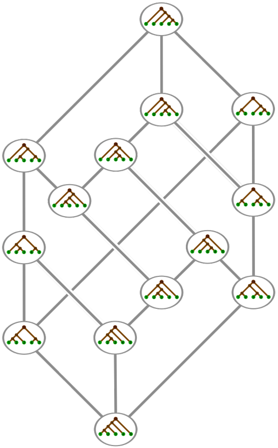
Tree rotation





- Suppose a vertex Q has left and right subtrees, with P being the root of the left subtree
- Right tree rotation $=$ A rotation at P moves P into Q's place and Q to the place of its right child
- Left tree rotation is defined similarly

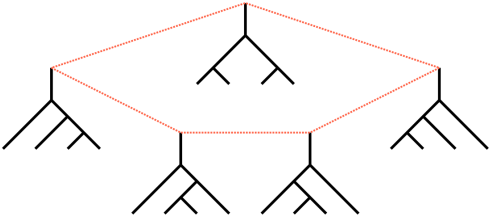
The Catalan numbers



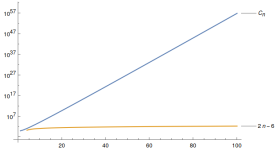
- Catalan numbers $= C_n = \frac{1}{n+1}\binom{2n}{n}$ Asymptotically $C_n \sim 4^n/(n^{3/2}\sqrt{\pi})$
- Catalan numbers count the number of binary trees with n vertices
- Question How difficult is it to relate the $\approx 4^n$ binary trees via rotation?

Enter, the theorem

All binary trees are rotation distance at most $2n-6$ from one another

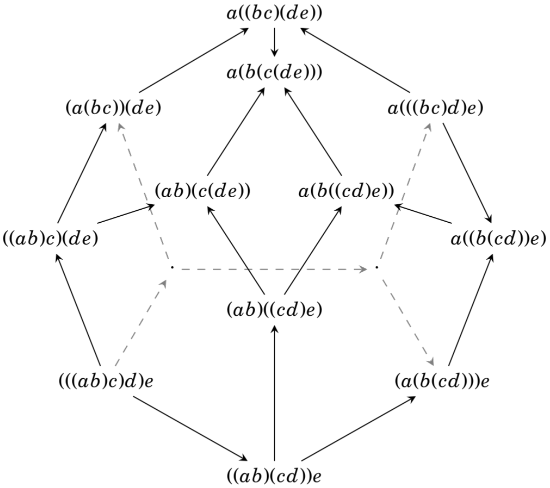


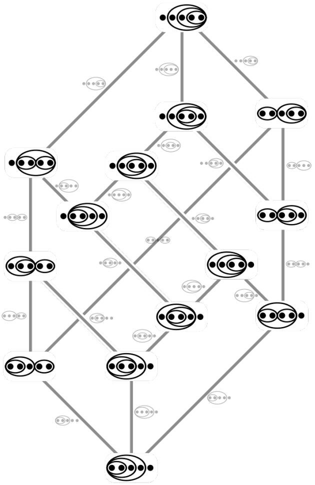
- Note how small $2n-6$ is compared to $C_n \sim 4^n/(n^{3/2}\sqrt{\pi})$



- The bound is in fact achieved for all $n \geq 11$

The associahedron





- The underlying graph is the associahedron
- This shows up everywhere – and the previous theorem tells us something about its size

Thank you for your attention!

I hope that was of some help.