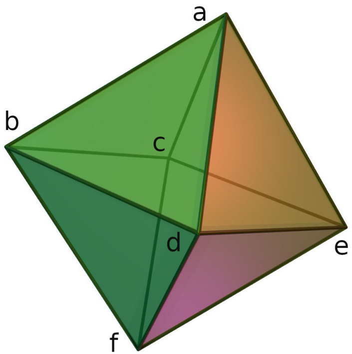
# What are...IBIS groups?

Or: Perfectly broken symmetries

## Symmetry groups



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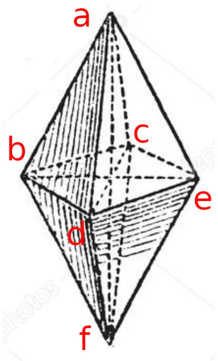
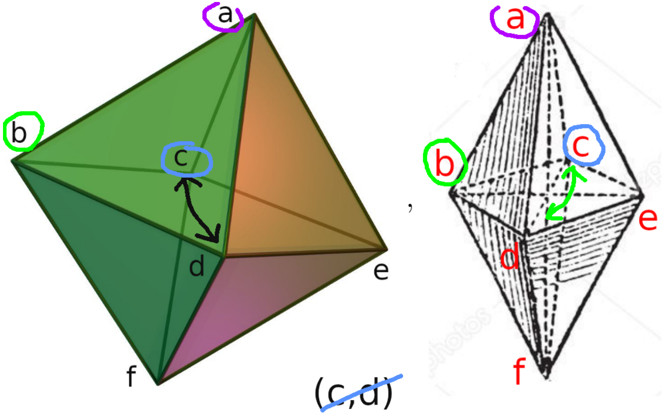



$$S(O) = \langle (a, f), (b, d)(c, e), (b, c, e, d) \rangle$$

- A permutation group $=$ subgroup of $S_n = \mathrm{Aut}(\{1, ..., n\})$ acting by permutation
- We think of these as symmetry groups of geometric objects
- Example Above we have the symmetry group of the irregular (“pulling top and bottom”, see right) octahedron

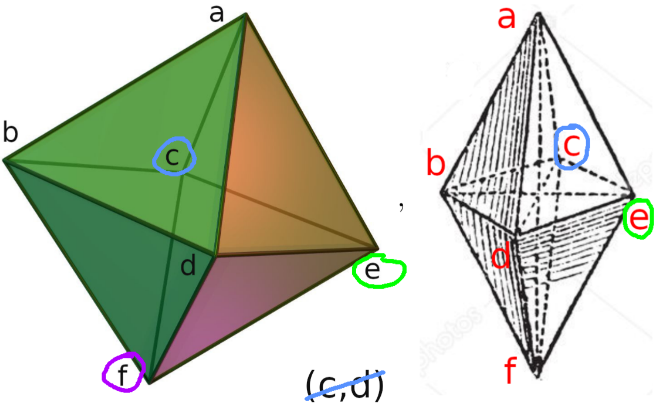
# Breaking symmetries




$$S(O) = \langle (a, f), (b, d)(c, e), (b, c, e, d) \rangle$$

- Fixing $a, b, c$ breaks all symmetries Call $[a, b, c]$ a(n ordered) basis
- $[a, b, c]$ is irredundant: no vertex can be omitted to break all symmetries
- To see this note that $(c, d) = (b, c, e, d) \circ (b, d)(c, e)$

# Breaking symmetries in perfection




$$S(O) = \langle (a, f), (b, d)(c, e), (b, c, e, d) \rangle$$

- $[f, e, c]$ also an irredundant basis
- In fact, all irredundant bases are of size three
- We call $G$ an IBIS group Irredundant Bases of Invariant Size

## Enter, the theorem

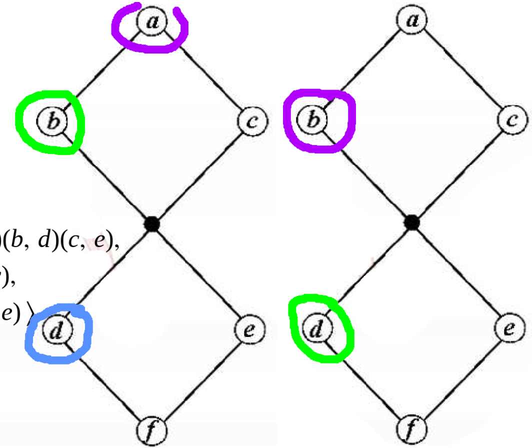
IBIS groups exist, but not all groups are IBIS

Moreover, a group is IBIS if and only if the bases come from a matroid

- Their classification is still open
- Which matroids arise is also still open

# Non IBIS groups

$$H = \langle h_1 = (a,\ f)(b,\ d)(c,\ e),\ h_2 = (b,\ c),\ h_3 = (d,\ e) \rangle$$




- $[a, b, d]$ is irredundant
- $[b, d]$ is also irredundant
- Hence, the group $H$ is not IBIS

**Thank you for your attention!**

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I hope that was of some help.