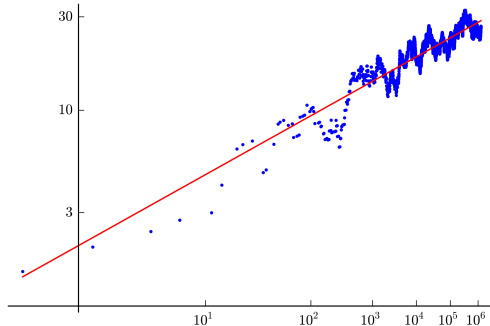


What are...computer conjectures?

Or: A computer can write poems, so...

Conjectures!



- ▶ Mathematics is, at least partially, about **good conjectures**
- ▶ Although it took a while to get started, computers are nowadays **key** for the art of conjecturing
- ▶ **Early and famous example** The Birch and Swinnerton-Dyer conjecture (millennium price problem) was discovered by computer
- ▶ In this video we discuss how machines can generate conjectures **“themselves”**

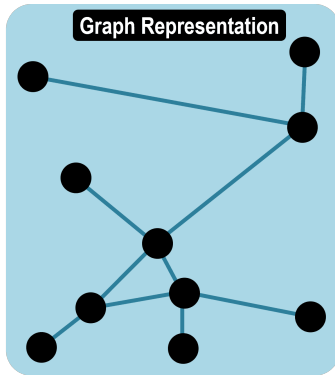
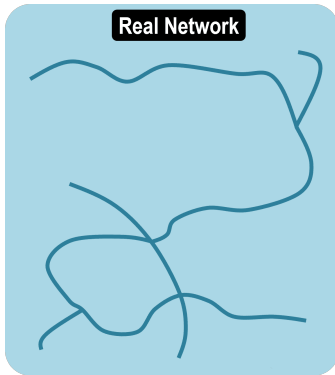
Stage 1: computer assisted conjectures

```
daniel@ubuntu:~/usr/local/magma/Decke-Hall$ magma -b type:=A4 I:=[1,2] Examples/BSBasis2.m
ld      1d      3      4      32      34      43      432      343      321      324      3214      3243      4321      3432      34321      32432      32143      324321      321432      3214321
1d      1
3      1
4      1
32     1
34     1
43     1
432    1
343    1
321    1
324    1
3214   1
3243   1
4321   1
3432   1
34321  1
32432  1
32143  1
324321 1
321432 1
3214321 1
```

```
daniel@ubuntu:~/usr/local/magma/Decke-Hall$ magma -b type:=A4 I:=[2,3] Examples/BSBasis2.m
ld      1d      4      12     14     43     123     124     143     432     1432     1234     1243     4321     12343     14321     12432     124321     123432     1234321
1d      1
4      1
12     1
14     1
43     1
123    1
124    1
143    1
432    1
1432   1
1234   1
1243   1
4321   1
12343  1
14321  1
12432  1
124321 1
123432 1
1234321 1
```

- ▶ Above A Magma calculation
- ▶ Conjectures are often born from calculations, e.g. from hand-calculate prime tables
- ▶ Since ~1950 computers have successively replaced by hand calculations and one gets more data for conjectures

Stage 2: computer generated conjectures



- ▶ **Bait** Graffiti (~1985) is a program that knows certain graphs and graph properties
- ▶ Graffiti creates conjectures by **data search**, trying to match graph+property
- ▶ **Catch** Most crucially, the setting is very restricted and almost all conjectures are rather boring
- ▶ Graffiti was the first of its kind but we have **many others** now

Enter, the theorem

Graffiti found the following nontrivial conjectures on the first run :

1. average temperature \leq rank
2. average temperature \leq variance of degree + maximal frequency of degree
3. inverse degree \leq Randic index + maximal frequency of degree
4. mode of distance \leq radius + Randic index
5. mode of distance \leq average distance + Randic index
6. mode of distance \leq matching + average distance
7. radius \leq zenith + maximal frequency of degree
8. radius \leq variance of degree + maximal frequency of degree
9. radius \leq Randic index + average temperature
10. radius \leq Randic index + variance of degree
11. radius \leq average distance + Randic index
12. radius \leq 1 + Randic index
13. average distance \leq Randic index
14. average distance \leq independence
15. average distance \leq variance of degree + maximal frequency of degree
16. average distance \leq variance of degree + inverse degree
17. average distance \leq mode of distance + inverse degree
18. chromatic number \leq 1 + rank

1. radius \leq independence
2. average temperature \leq chromatic number

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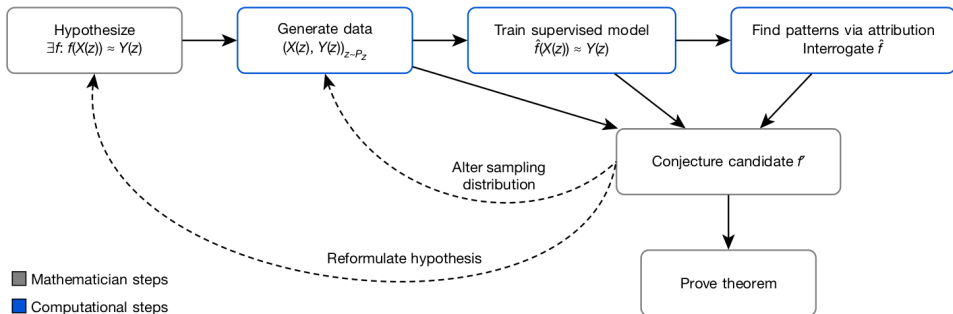
S. Fajtlowicz

3. diameter \leq rank
4. radius \leq matching
5. average distance \leq 1 + matching
6. inverse degree \leq independence + Randic index
7. inverse degree \leq matching + independence
8. Randic index \leq matching + independence
9. Randic index \leq number of vertices
10. average degree \leq matching + Randic index
11. Randic index \leq matching + inverse degree
12. matching \leq Randic index + inverse degree

The conjectures on the right have been proven quickly

The author of Graffiti states: "None of these theorems are very difficult, but almost every one, I think, is of some interest"

Stage 3: computer guided conjectures



- ▶ **Idea** Machine learning has tools that can effectively detect patterns in data
- ▶ Hypothesize $\exists f$, then **estimate** a function \hat{f} and try to find a closed form f' – if \hat{f} is better than expected by chance, there might be such a relationship
- ▶ These machine learning methods already gave new discoveries in **topology and in representation theory**

Thank you for your attention!

I hope that was of some help.