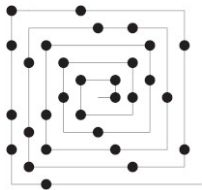


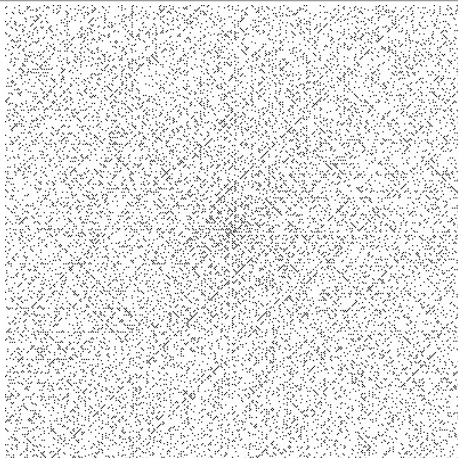
What is...the prime number theorem?

Or: Let us not count!

Primes are rather random



101	100	99	98	97	96	95	94	93	92	91
102	65	64	63	62	61	60	59	58	57	90
103	66	57	36	35	34	33	32	31	56	89
104	67	38	17	16	15	14	13	30	55	88
105	68	39	18	5	4	3	12	29	54	87
106	69	40	19	6	1	2	11	28	33	86
107	70	41	20	7	8	9	10	27	52	85
108	71	42	21	22	23	24	25	26	51	84
109	72	43	44	45	46	47	48	49	50	83
110	73	74	75	76	77	78	79	80	81	82
111	112	113	114	115	116	117	118	119	120	121



- ▶ Prime numbers appear essentially randomly
- ▶ Zooming out, they mostly look like noise
- ▶ Question Can we say anything about when they pop-up?

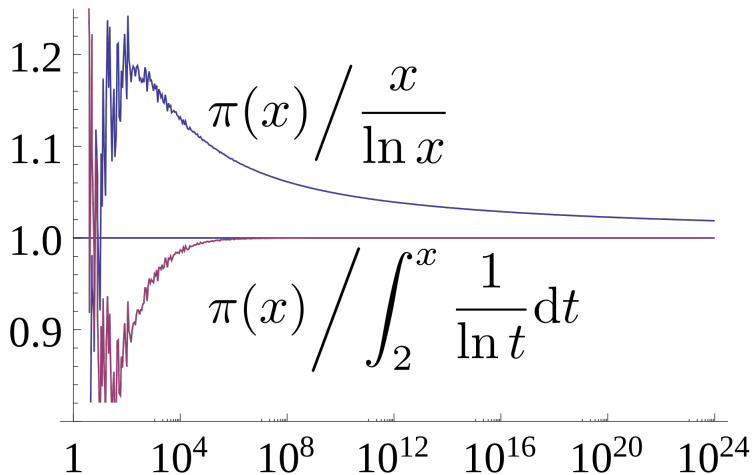
Counting primes

Limite x	Nombre y		Limite x	Nombre y	
	par la formule.	par les Tables.		par la formule.	par les Tables.
10000	1230	1230	100000	9588	9592
20000	2268	2263	150000	13844	13849
30000	3252	3246	200000	17982	17984
40000	4205	4204	250000	22035	22045
50000	5136	5134	300000	26023	25998
60000	6049	6058	350000	29961	29977
70000	6949	6936	400000	33854	33861
80000	7838	7837			
90000	8717	8713			

Actually, #primes < 1000
 = 1229...

- ▶ Counting primes is difficult
- ▶ Legendre and others (~1793) counted primes up to 400000 and more

Not counting primes



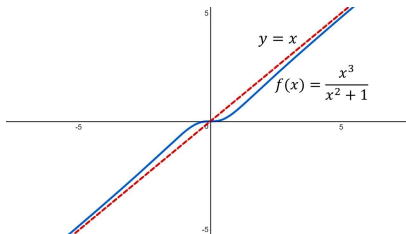
- ▶ Counting primes is **difficult** but...
- ▶ ... an **asymptotic formula** is not so difficult to guess

Enter, the theorem

For $\pi(n)$ = number of primes $\leq n$ we have

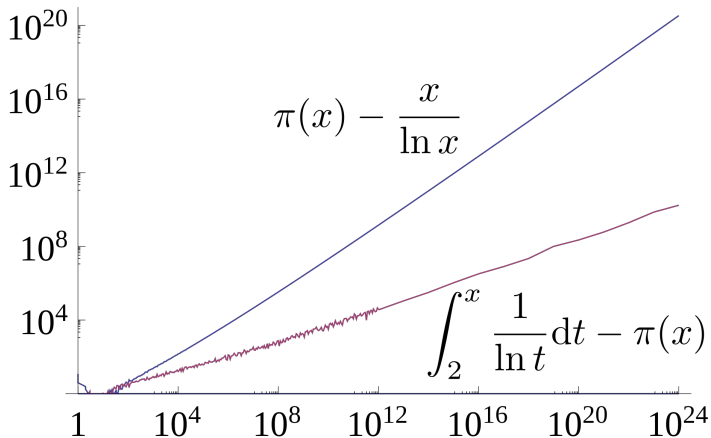
$$\pi(n) \sim n/\log(n)$$

- ▶ **Upshot** $n/\log(n)$ is super easy to compute
- ▶ \sim = asymptotically, *i.e.* for n large we have $\pi(n)$ “=” $n/\log(n)$



- ▶ This theorem has **many proofs** (some collect such proofs)
- ▶ There is also a version using the logarithmic integral $Li(n)$

Careful with absolute errors



- ▶ $\pi(n) \sim n/\log(n)$ does not imply that $|\pi(n) - n/\log(n)|$ is small for large n
- ▶ In fact, $|\pi(n) - n/\log(n)|$ gets arbitrary large
- ▶ $\pi(n) - Li(n)$ switches signs infinitely often

Thank you for your attention!

I hope that was of some help.