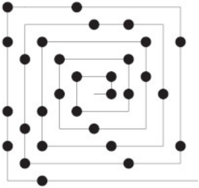
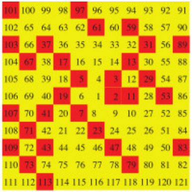
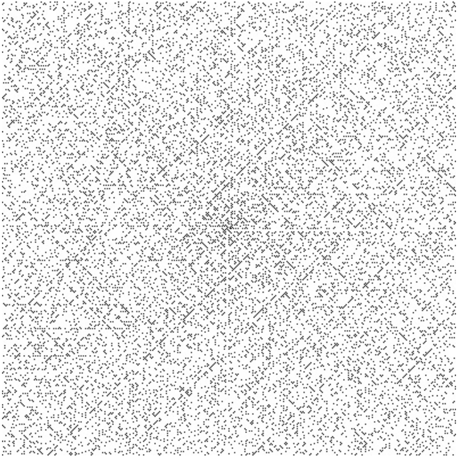
What is...the prime number theorem?

Or: Let us not count!

Primes are rather random







- Prime numbers appear essentially randomly
- Zooming out, they mostly look like noise
- Question Can we say anything about when they pop-up?

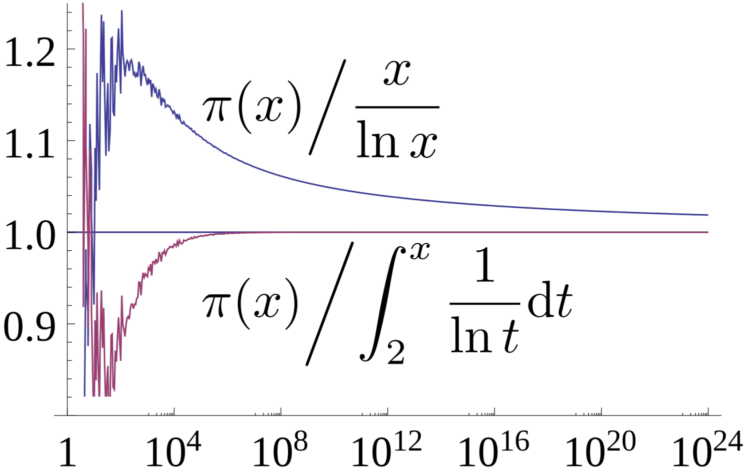
Counting primes

Limite x	Nombre y par la formule.	Nombre y par les Tables.	Limite x	Nombre y par la formule.	Nombre y par les Tables.
10000	1230	1230	100000	9588	9592
20000	2268	2263	150000	13844	13849
30000	3252	3246	200000	17982	17984
40000	4205	4204	250000	22035	22045
50000	5136	5134	300000	26023	25998
60000	6049	6058	350000	29961	29977
70000	6949	6936	400000	33854	33861
80000	7838	7837			
90000	8717	8713			

Acctually, #primes<1000 =1229...

- Counting primes is difficult
- **Legendre and others (∼1793)** counted primes up to 400000 and more

Not counting primes



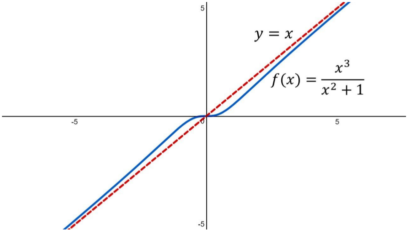
- Counting primes is difficult but...
- ... an asymptotic formula is not so difficult to guess

Enter, the theorem

For $\pi(n)$ = number of primes $\leq n$ we have

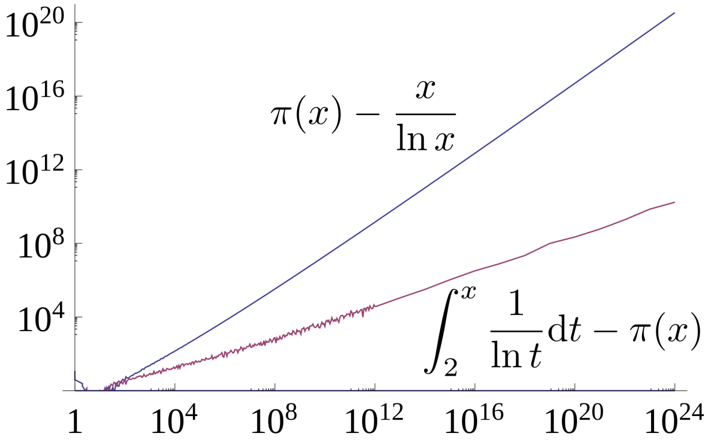
$$\pi(n) \sim n/\log(n)$$

- Upshot $n/\log(n)$ is super easy to compute
- $\sim$ = asymptotically, *i.e.* for n large we have $\pi(n)$ "=" $n/\log(n)$



- This theorem has many proofs (some collect such proofs)
- There is also a version using the logarithmic integral $Li(n)$

Careful with absolute errors



- $\pi(n) \sim n/\log(n)$ does not imply that $|\pi(n) - n/\log(n)|$ is small for large n
- In fact, $|\pi(n) - n/\log(n)|$ gets arbitrary large
- $\pi(n) - Li(n)$ switches signs infinitely often

Thank you for your attention!

I hope that was of some help.