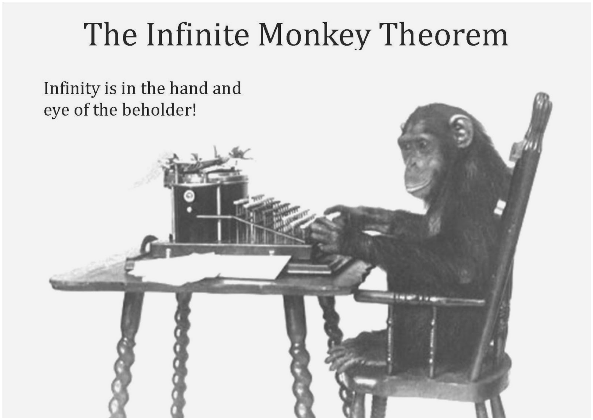
What is...the balanced graph theorem?

Or: Sudden appearance

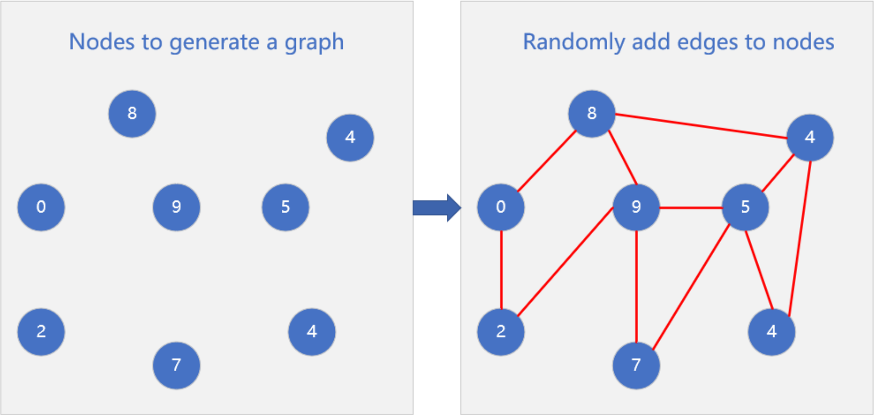
The "essence" of randomness



The monkey seriously needs a more modern device

- What is random? Guess: random = everything that can happen happens eventually
- Analogy A monkey would almost surely type every possible finite text an infinite number of times
- Let us test the guess in graph theory!

Random graphs

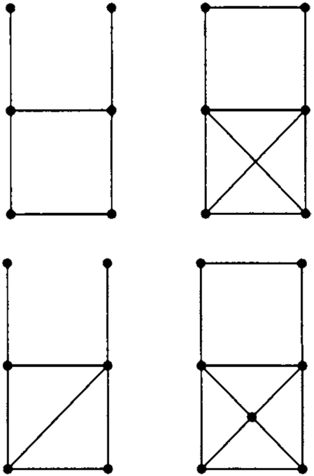


- In this video random graph = coin flip graph $G_{n,p}$ on n vertices
- $G_{n,p}$ = flip a (biased) coin for every pair of vertices $v \neq w$
- Put an edge $v - w$ with probability $0 \leq p \leq 1$

The target: balanced graphs

balanced:

not balanced:



- ▶ A graph is **balanced** if no subgraph of it has > average degree $d = |E|/|V|$
- ▶ **Example** Complete graphs, cycles and trees are balanced
- ▶ **Question** How like appears $B_{k,l}$ (balanced with k vertices and l edges) in $G_{n,p}$?

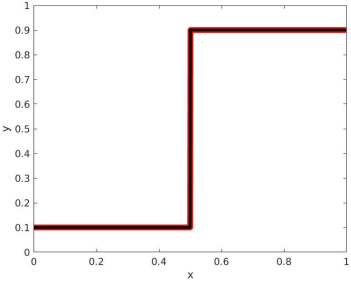
Enter, the theorem

Let $k \geq 2$ and $k-1 \leq l \leq k(k-1)/2$

(1) If $p(n)n^{k/l} \rightarrow_{n\rightarrow\infty} 0$, then almost no $G_{n,p}$ contains a $B_{k,l}$

(2) If $p(n)n^{k/l} \rightarrow_{n\rightarrow\infty} \infty$, then almost all $G_{n,p}$ contain a $B_{k,l}$

- We allow p to vary with n
- Example $p(n) = 0.5$, then "almost all" applies
- Example If $p(n) = 1/n^2$, then the $B_{k,l}$ appear rather suddenly



Random is still random

- "Random = everything that can happen happens eventually" still works
- But when one varies the probability there might be noncontinuous behavior
- Question What happens if the monkey hits 'G' with probability $p \to 0$?

Thank you for your attention!

I hope that was of some help.