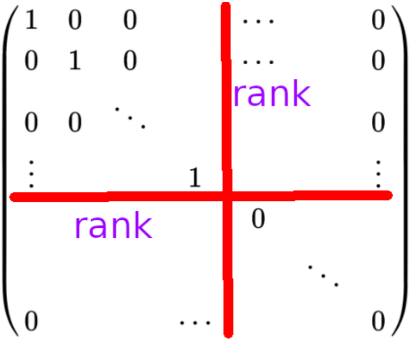
What is...the matrix equivalence theorem?

Or: One is ok. Two is, well... Three is impossible

$m = 1$ is easy



- Matrices $A = (A_1, ..., A_m)$ and $B = (B_1, ..., B_m)$ are simultaneously equivalent if:

$$(A \sim B) \Leftrightarrow (\exists P, Q : \forall i : A_i = Q^{-1} B_i P \text{ with } P, Q \text{ invertible })$$

  Crucial: There is only one P and one Q

- Question How can we classify equivalent matrices, say over $\mathbb{C}$?
- For $m = 1$ the classification is given by the rank (known for a long time)

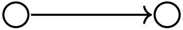
$m = 2$ is doable but "ugly"

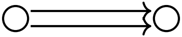
$$J_j(\alpha) \equiv \begin{bmatrix} \alpha-\lambda & 1 & & \\ & \cdot & \cdot & \\ & & \cdot & 1 \\ & & & \alpha-\lambda \end{bmatrix} \quad \text{and} \quad N_j \equiv \begin{bmatrix} 1 & -\lambda & & \\ & \cdot & \cdot & \\ & & \cdot & -\lambda \\ & & & 1 \end{bmatrix}$$

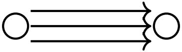
$$L_j \equiv \begin{bmatrix} -\lambda & 1 & & \\ & \cdot & \cdot & \\ & & -\lambda & 1 \end{bmatrix} \quad \text{and} \quad L_j^T \equiv \begin{bmatrix} -\lambda & & \\ 1 & \cdot & \\ & \cdot & -\lambda \\ & & 1 \end{bmatrix}$$

- For $m = 2$ a normal form is given by Kronecker's canonical form
- Kronecker's normal form is similar to the Jordan normal form, but with four different blocks
- For $m = 2$ the classification is thus given by finitely many discrete parameters = sizes and types of blocks; and two continuous parameters = "eigenvalues" α and λ

$m = 3$ is impossible

$m = 1$: 

$m = 2$: 

$m = 3$: 

- The problem of simultaneous equivalence can be associated to a quiver
- Quiver = directed graph "It contains arrows"
- One then can formally prove that $m = 3$ is "impossible"

Enter, the theorem

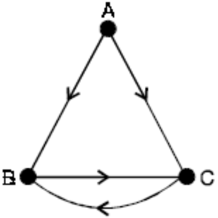
A matrix problem associated to a finite connected quiver Q without oriented cycles is...

(1) ...finite if and only if $\overline{Q}$ is of ADE type

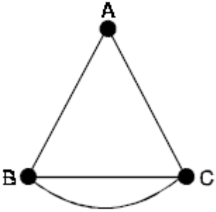
(2) ...infinite tame if and only if $\overline{Q}$ is of affine ADE type

(3) ...wild otherwise

- Finite = classification is given by finitely many discrete parameters; infinite tame = finitely many discrete and continuous parameters; wild = forget it
- Q = the quiver; $\overline{Q}$ = the underlying graph

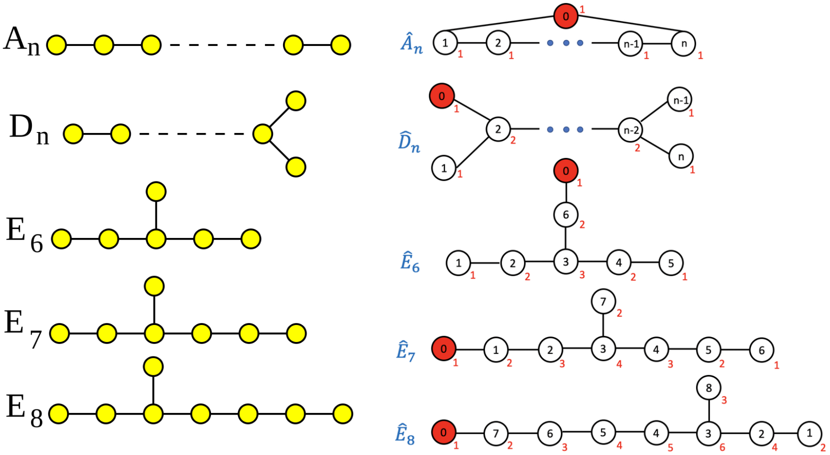


Q



$\overline{Q}$

ADE and all that



- ADE graphs and friends appear everywhere
- Left The ADE types; Right The affine ADE types

Thank you for your attention!

I hope that was of some help.