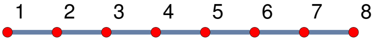
What is...an ADE classification?

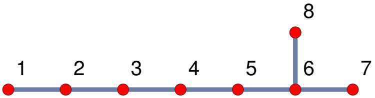
Or: A strange reoccurring pattern

Problem. The graphs with spectral radius $\rho < 2$ are...?



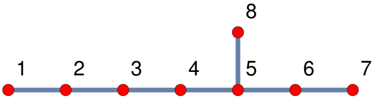
$$\leftrightsquigarrow \begin{pmatrix} 0&1&0&0&0&0&0&0\\ 1&0&1&0&0&0&0&0\\ 0&1&0&1&0&0&0&0\\ 0&0&1&0&1&0&0&0\\ 0&0&0&1&0&1&0&0\\ 0&0&0&0&1&0&1&0\\ 0&0&0&0&0&1&0&1\\ 0&0&0&0&0&0&1&0 \end{pmatrix}$$

$\rho \approx 1.88$



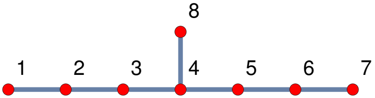
$$\leftrightsquigarrow \begin{pmatrix} 0&1&0&0&0&0&0&0\\ 1&0&1&0&0&0&0&0\\ 0&1&0&1&0&0&0&0\\ 0&0&1&0&1&0&0&0\\ 0&0&0&1&0&1&0&0\\ 0&0&0&0&1&0&1&1\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0&0 \end{pmatrix}$$

$\rho \approx 1.95$



$$\leftrightsquigarrow \begin{pmatrix} 0&1&0&0&0&0&0&0\\ 1&0&1&0&0&0&0&0\\ 0&1&0&1&0&0&0&0\\ 0&0&1&0&1&0&0&0\\ 0&0&0&1&0&1&0&1\\ 0&0&0&0&1&0&1&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&1&0&0&0 \end{pmatrix}$$

$\rho \approx 1.99$



$$\leftrightsquigarrow \begin{pmatrix} 0&1&0&0&0&0&0&0\\ 1&0&1&0&0&0&0&0\\ 0&1&0&1&0&0&0&0\\ 0&0&1&0&1&0&0&1\\ 0&0&0&1&0&1&0&0\\ 0&0&0&0&1&0&1&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&1&0&0&0&0 \end{pmatrix}$$

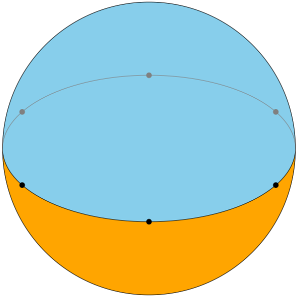
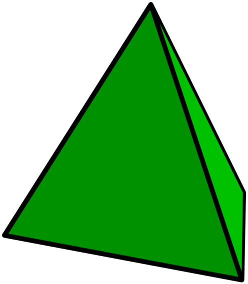
$\rho = 2$

The spectral radius ρ a.k.a. maximal eigenvalue is a measurement for complexity

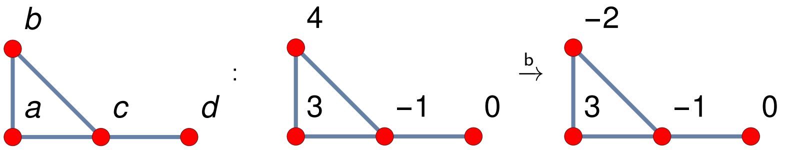
Problem. The finite subgroups of $\mathrm{SO}(3)$ are...?

$\mathrm{SO}(3)$ = orthogonal 3x3 matrices. Let α, β, γ and δ be a regular n-gon, a tetrahedron, a cube/octahedron, and a dodecahedron/icosahedron

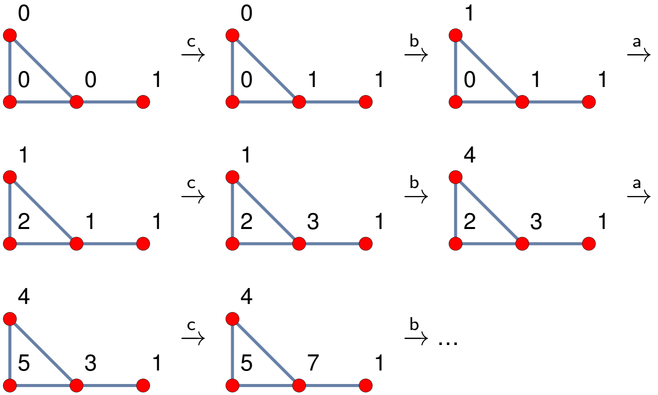
$$A_{n-1} = \{\rho \in \mathrm{SO}(2) \mid \rho \text{ preserves } \alpha\}$$
$$D_{n+2} = \{\rho \in \mathrm{SO}(3) \mid \rho \text{ preserves } \alpha\}$$
$$E_6 = \{\rho \in \mathrm{SO}(3) \mid \rho \text{ preserves } \beta\}$$
$$E_7 = \{\rho \in \mathrm{SO}(3) \mid \rho \text{ preserves } \gamma\}$$
$$E_8 = \{\rho \in \mathrm{SO}(3) \mid \rho \text{ preserves } \delta\} \text{...more?}$$

$D_8 \leftrightsquigarrow$  , $E_6 \leftrightsquigarrow$ 

Problem. Population stability arises for what...?



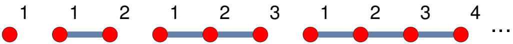
This one is not stable:



Enter, the theorem/philosophy!

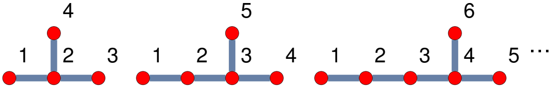
All of the three problems (spectral radius of graphs, finite subgroups of $\mathrm{SU}(2)$, population stability) have the same answer: the ADE classification!

(a) Type A graphs (infinitely many)



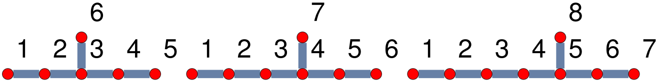
(continues)

(b) Type D graphs (infinitely many)



(continues)

(c) Type E graphs, E_6, E_7 and E_8 (finitely many)



(stops)

Many seemingly not connected question have this (or a related) type of answer!

More fun with ADE

- Simple Lie groups/algebras Lie theory
- Surface singularities Algebraic geometry
- Elementary catastrophes Dynamical systems
- Quivers of finite type Representation theory
- Minimal models of 2d conformal field theory Quantum field theory
- 4d $\mathcal{N} = 2$ superconformal gauge quiver theories Quantum field theory
- Simply laced finite Coxeter types Geometry
- Positive definite quadratic forms on graphs Combinatorics
- Many more...

Thank you for your attention!

I hope that was of some help.