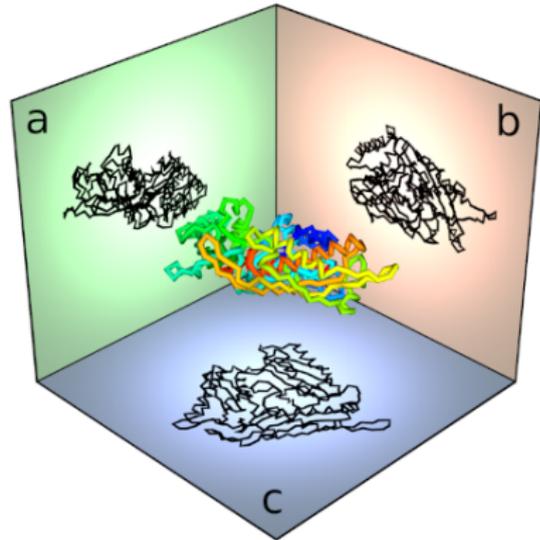
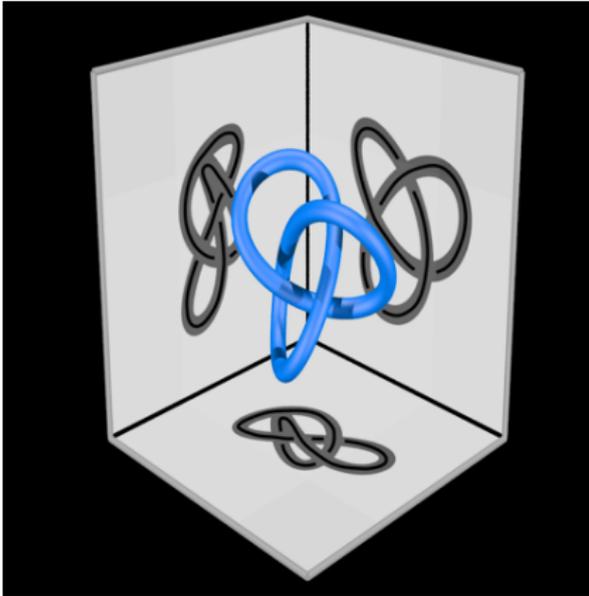


**What are...skein relations?**

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Or: Life knows somehow...

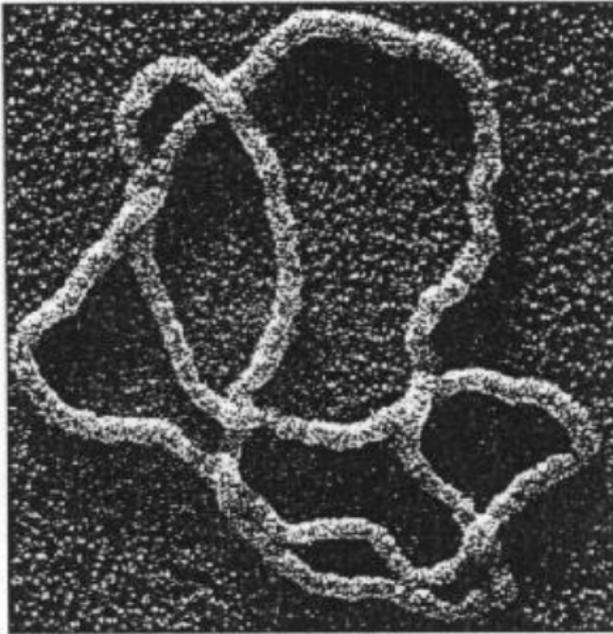
## Knots again



- ▶ A knot is a closed string (a circle  $S^1$ ) in three spaces
- ▶ Knots are studied by projections to the plane **Shadows**
- ▶ The main goal of knot theory is to find **invariants**

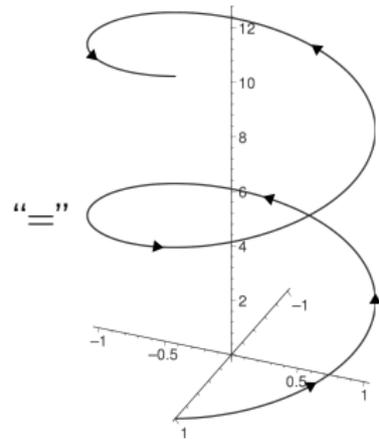
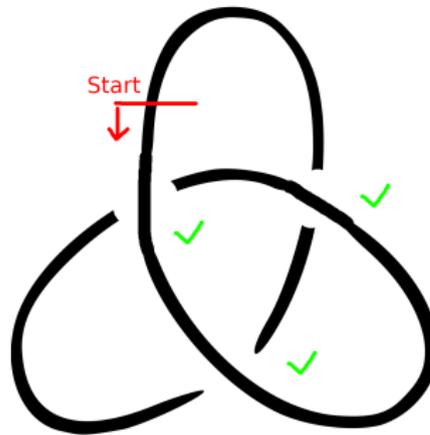
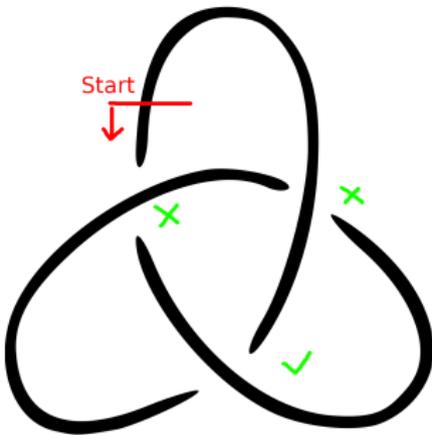
## The approach of life

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- 
- ▶ DNA is often **knotted**; or reproduction DNA needs to be **unknotted**
  - ▶ One of the main unknotting enzymes do the above **crossing swap**
  - ▶ Math should be able to explore this ;-)  
careful: this is historically backwards

# Crossing changes can undo knotted DNA



- ▶ **Theorem** Flipping crossing can trivialize every link = knot with multiple components
- ▶ **Proof** Produce a helix
- ▶ **Life knows what its doing!**

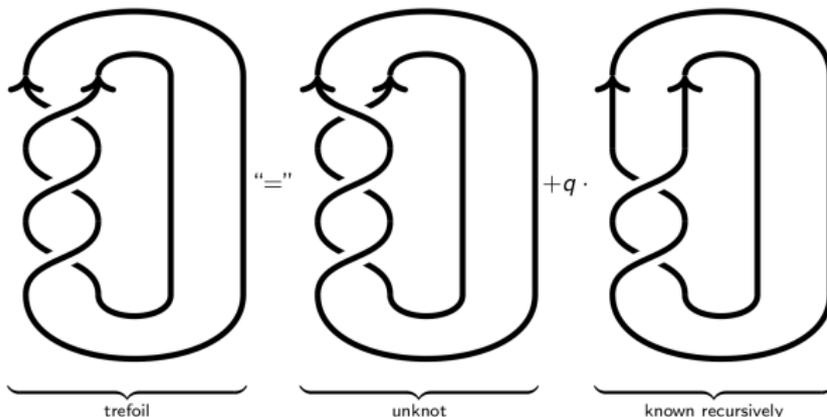
# Enter, the theorem

A skein relation, i.e. a relation of the form

$$A \cdot \begin{array}{c} \nearrow \\ \searrow \\ \nearrow \\ \searrow \end{array} = B \cdot \begin{array}{c} \nearrow \\ \searrow \\ \nearrow \\ \searrow \end{array} + C \cdot \begin{array}{c} \uparrow \\ \uparrow \end{array}$$

will, applied correctly, produce a linear combination of unlinks

► **Example.** For  $A = 1$ ,  $B = 1$  and  $C = q$ :



► This goes back to Alexander  $\sim 1928$ , but life wins by a few million years ;-)

## Almost all coefficients are useless

General:  $A \cdot \begin{array}{c} \swarrow \nearrow \\ \nearrow \swarrow \end{array} = B \cdot \begin{array}{c} \swarrow \nearrow \\ \nearrow \swarrow \end{array} + C \cdot \begin{array}{c} \uparrow \\ \uparrow \end{array}$

Life:  $1 \cdot \begin{array}{c} \swarrow \nearrow \\ \nearrow \swarrow \end{array} = 1 \cdot \begin{array}{c} \swarrow \nearrow \\ \nearrow \swarrow \end{array} + 0 \cdot \begin{array}{c} \uparrow \\ \uparrow \end{array}$

Alexander:  $1 \cdot \begin{array}{c} \swarrow \nearrow \\ \nearrow \swarrow \end{array} = 1 \cdot \begin{array}{c} \swarrow \nearrow \\ \nearrow \swarrow \end{array} + q \cdot \begin{array}{c} \uparrow \\ \uparrow \end{array}$

Jones:  $q^{-1} \cdot \begin{array}{c} \swarrow \nearrow \\ \nearrow \swarrow \end{array} = q \cdot \begin{array}{c} \swarrow \nearrow \\ \nearrow \swarrow \end{array} + (q^{1/2} - q^{-1/2}) \cdot \begin{array}{c} \uparrow \\ \uparrow \end{array}$

- ▶ Most (=99.9% by experience) choices for  $A, B, C$  won't work (need to satisfy Reidemeister moves and well-definedness)
- ▶ Most (=99.9% by experience) legit choices for  $A, B, C$  give life or Alexander
- ▶ Other choices were found "by miracle"

**Thank you for your attention!**

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I hope that was of some help.