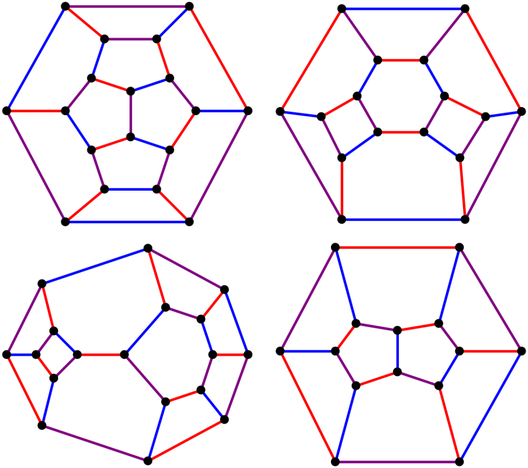
What is...counting of Tait colorings?

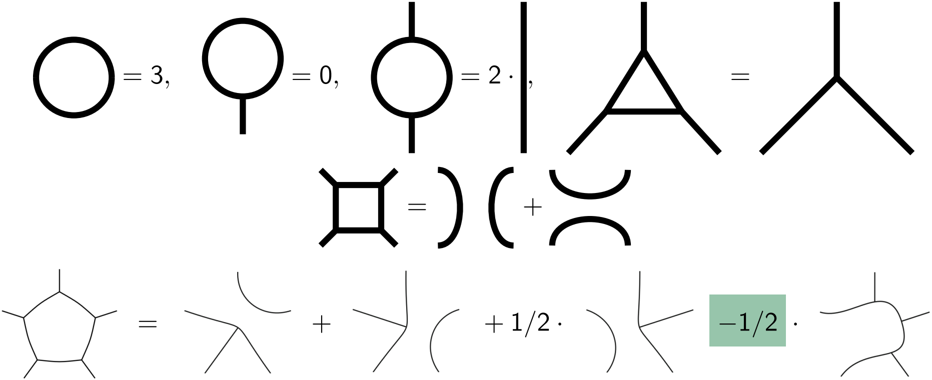
Or: Evaluating graphs

Webs and Tait colorings



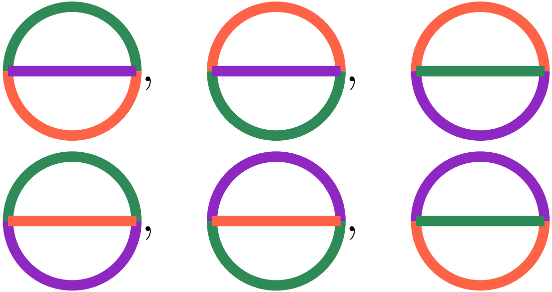
- Web = planar trivalent
- Tait coloring = coloring of the edges of a web with three colors
- Question How can we count the number of Tait colorings?

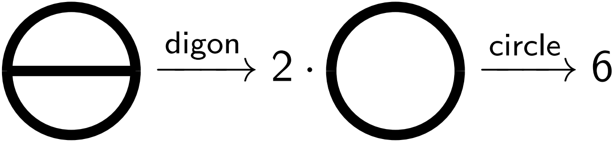
Evaluation rules



- We allow linear combinations of webs
- We apply the above rules recursively to simplify webs
- Each step makes the web simpler so the recursion will terminate

Webs to numbers





- The theta has 6 Tait colorings
- The theta evaluates to 6
- Question Is that a coincidence?

Enter, the theorem

The evaluation of webs $ev(w)$...

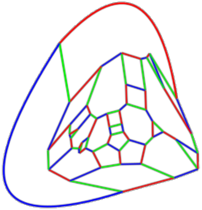
- ...is well-defined (*i.e.* doesn't depend on the face bursting order)
- ...always terminates in a number
- ...counts the number of Tait colorings

Thus, showing for w bridgeless that

$$ev(w) \in \mathbb{Z}_{\geq 1}$$

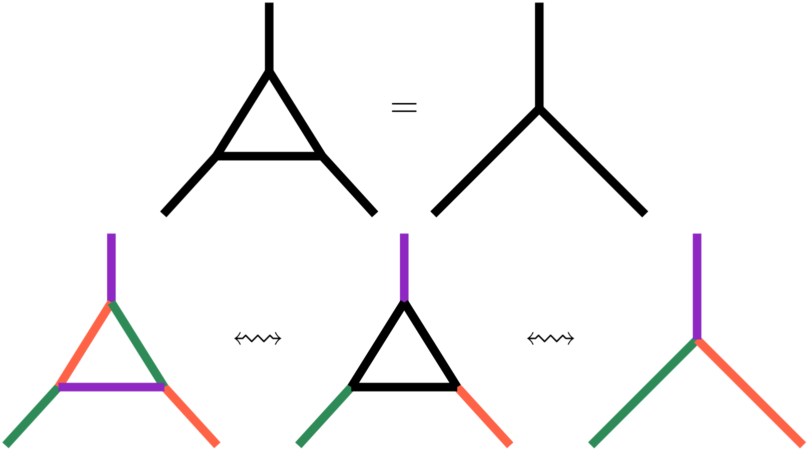
is equivalent to the four color theorem

- Fact Every web contains at least one n-gon for $n \leq 5$, *e.g.*



- The webs correspond to intertwiners of $SO(3)$

Sketch of the proof



- If we know the boundary color of the triangle, then there is a no way or an unique way to fill in colors
- In other words, both side have the same number of Tait colorings
- One checks the same for the other relations

Thank you for your attention!

I hope that was of some help.