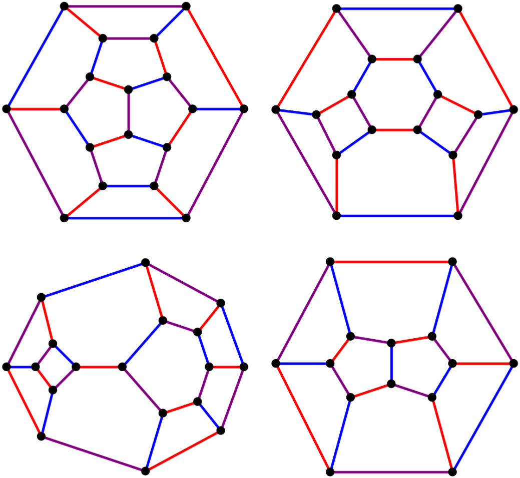
What is...a Tait coloring?

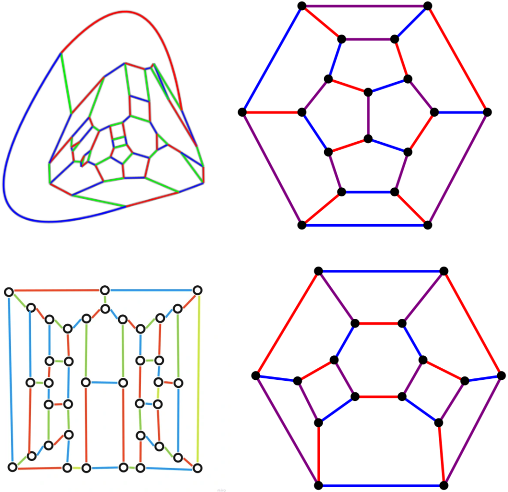
Or: Towards the four color theorem

Webs and graphs



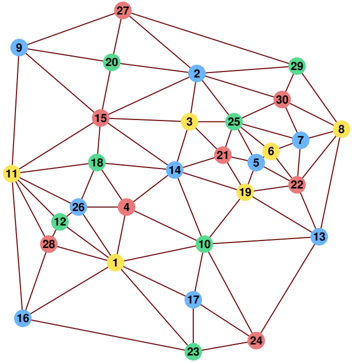
- We take planar trivalent graphs; I call them webs
- Planar = can be drawn in the plane without crossings
- Trivalent = every vertex has degree 3

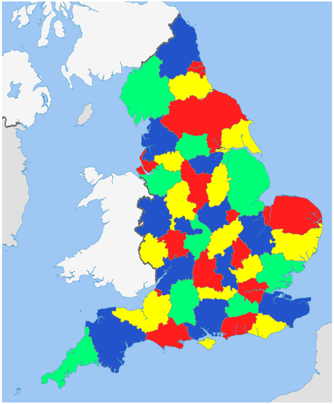
Web colorings



- Tait coloring = three coloring of the edges of a web
- All edges adjacent to a vertex get different colors using three colors only

Now something completely different...or?





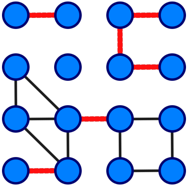
- Map coloring = adjacent countries get different colors
- Four colors suffice (4CT) Every planar graph is four-colorable
- Conjectured by Francis Guthrie ∼1852 (counties of England)
- Open for more than 100 years; known proofs are complicated
- Question How is that related to web coloring?

Enter, the theorem

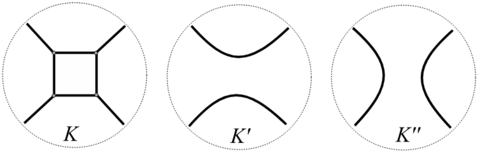
The 4CT is equivalent to 'every planar bridgeless web is 3-edge-colorable'

- See next slide for a sketch of a proof
- Bridgeless = no bridges; bridge = edges whose removal disconnect the graph

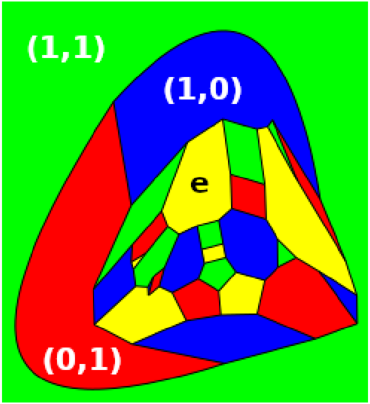
black = not a bridge
red = bridge
:

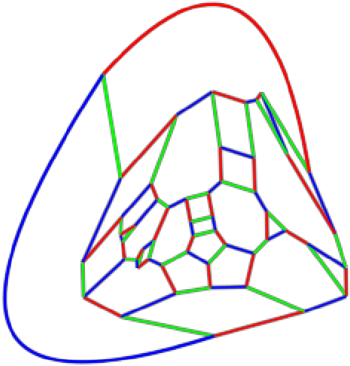


- In a follow-up video I show you an algorithm to count Tait colorings



From four colors to Tait colors and back





- To go from a planar graph to a web, triangulate the graph and take the dual
- Think of the four colors as elements of Klein's four group $\mathbb{Z}/2\mathbb{Z}\times\mathbb{Z}/2\mathbb{Z}$
- Multiply the elements of two adjacent faces to get the color for the edge
- For the way back use two-color chains

Thank you for your attention!

I hope that was of some help.