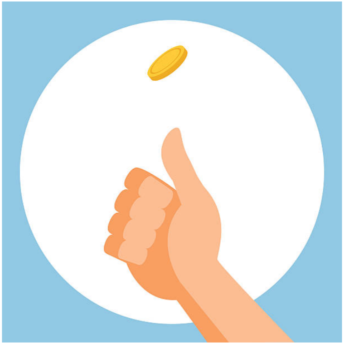
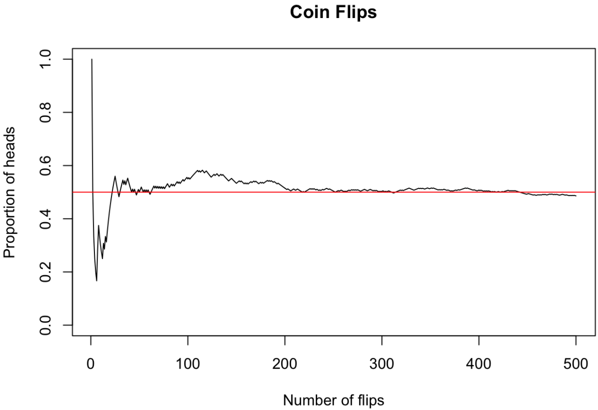
What is...the Rado graph?

Or: The law of large numbers for graphs

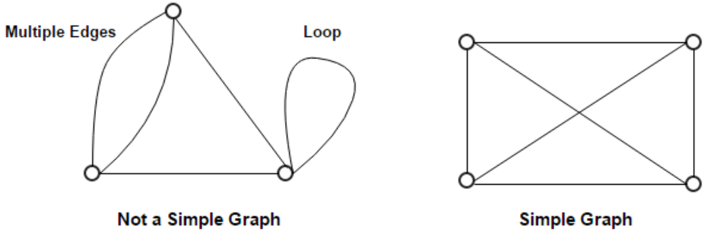
Law of large numbers

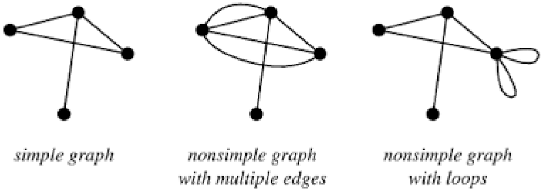




- Law of large numbers "=" flukes do not exist at infinity
- Tossing a (fair) coin very often and heads will show up 50% of the time
- In other words, all coin toss experiments are the same at infinity

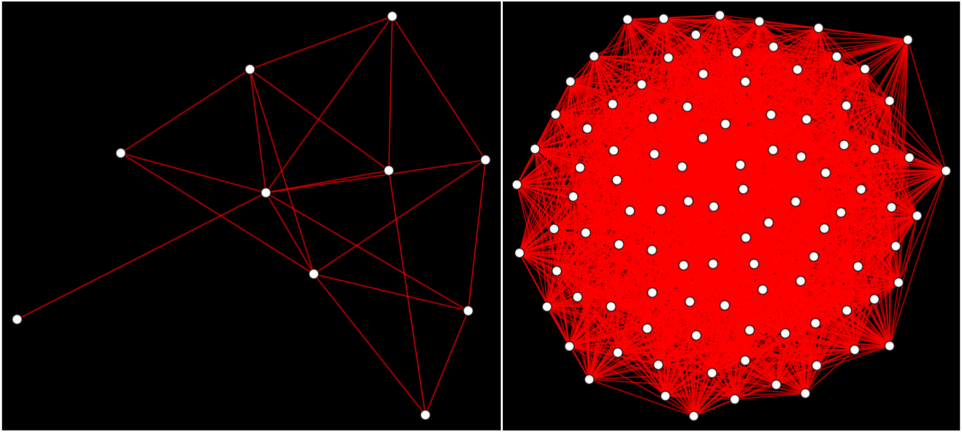
Random simple graphs





- Simple graph = a graph without multiple edges or loops
- Random (simple) graph = for each pair v, w of vertices with $v \neq w$ toss coin to decide whether we put an edge or not

Very large random graphs

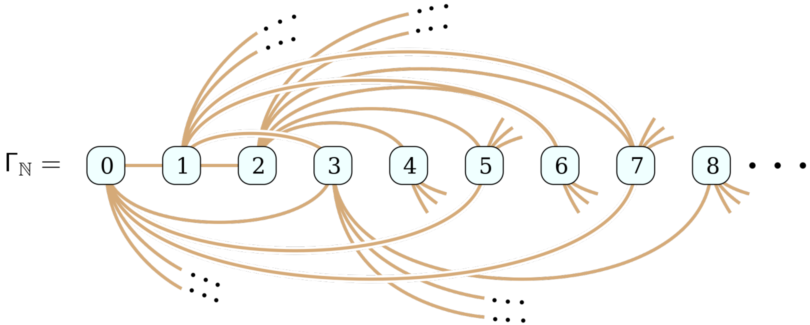


- Take a random simple graph Γ_n with n vertices and let n grow
- Some patterns seem to stabilize
- Question What happens in the limit $n = \infty$?

Enter, the theorem

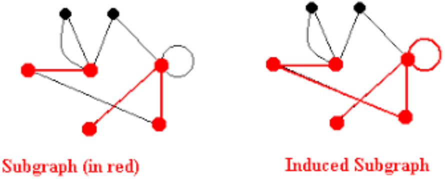
Two coin toss random graphs on $\mathbb{N}$ are almost surely isomorphic

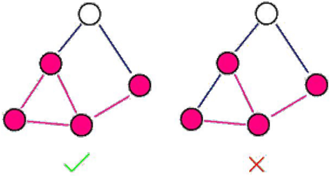
Thus, there is a unique infinite random graph $\Gamma_{\mathbb{N}}$



- Almost surely = with probability 1 (careful: infinity is around)
- The above justifies the alternative name "the random graph"
- Law of large numbers for graphs ∞ coin tosses will give the same end result

Finite graphs are free





- Induced subgraph = subgraph obtained by choosing certain vertices and all edges for these vertices
- Theorem Every finite graph is an induced subgraph of $\Gamma_{\mathbb{N}}$

Thank you for your attention!

I hope that was of some help.