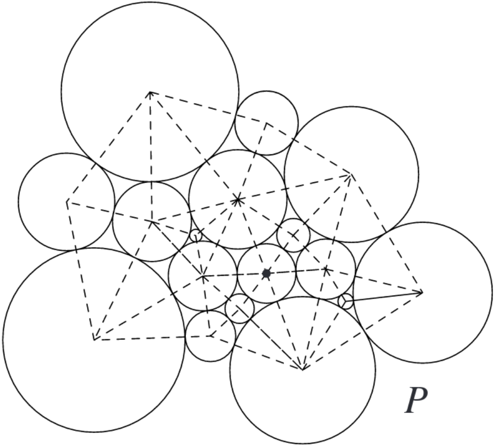
What is...the circle packing theorem?

Or: From circles to graphs and back

Citrus fruits packing

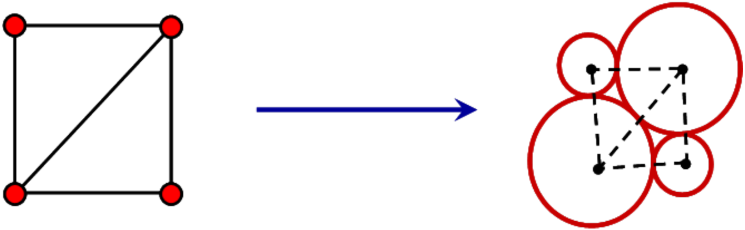
- Circle packing = arrangement of circles without overlaps and without possible enlargement
- In this video we put finitely many circles in the plane
- Question Is there a way to generate such packings?

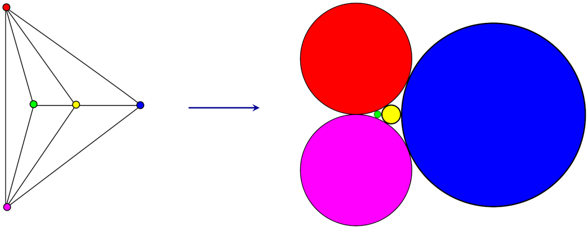
From a circle packing to a graph



- Intersection graph = vertices are midpoints of the circles, edges whenever circles touch
- From packings to graphs

From graph to a circle packing





- For a given type of graph (to be identified) we can go backwards by reversing the process from the previous slide
- From graphs to packings

Enter, the theorem

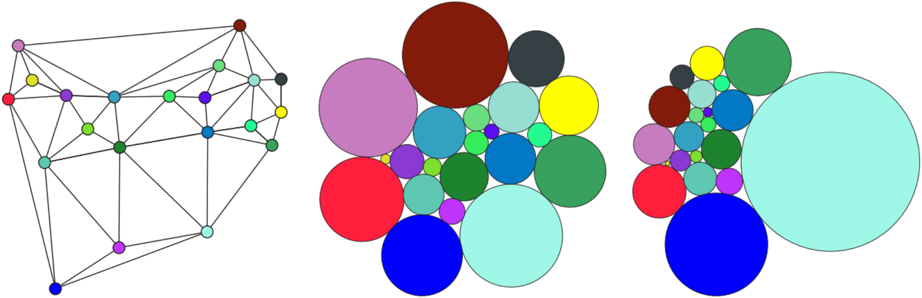
We have:

(i) For every circle packing there is connected simple planar graph

(ii) For every connected simple planar graph there is a circle packing

The connection is by intersection graphs

- There are algorithms to go from planar graphs to packings



- This is a useful tool to study various problems in planar geometry, conformal mappings and planar graphs

Geometry helps to understand graphs

- Penny graphs = graphs for which we get a circle packing with equal radii
- Penny graphs form a very nice class of graphs, *e.g.* for these the four color theorem is reasonably easy to prove

Thank you for your attention!

I hope that was of some help.